NOTES FOR

Twist 'til we tear the house down!

By James E. Beichler

1. William Kingdon Clifford, *The Common Sense of the Exact Sciences*, edited by Karl Pearson, newly edited with an introduction by James R. Newman and preface by Bertrand Russell, (New York: Dover, 1955; Reprint of the 1946 Knopf edition; Unaltered reprint of the third edition of 1899; First English edition, London: Macmillan, 1885; First American edition, New York: Appleton, 1885). See Chapter on "Position," pp.184-204.

2. William K. Clifford, "On the Space-Theory of Matter," presented 2 February 1870, *Transactions of the Cambridge Philosophical Society*, 1866/1876, 2: 157-158; Reprinted in William K. Clifford, Mathematical Papers, edited by Robert Tucker, with an introduction by H.J.S. Smith, (New York: Chelsea, 1968; An unaltered reproduction of the 1882 original): 21-22.

3. Sir Arthur S. Eddington, *Space, Time, and Gravitation*, (Cambridge: at the University Press, 1921), on p.192; E.T. Bell, *The Development of Mathematics*, second edition, (New York: McGraw-Hill, 1945; Reprint of the 1940 original), on p.360; Lloyd S. Swenson, The Genesis of Relativity, (New York: Burt Franklin, 1979), on p.36. At least in Eddington's case, the qualifiers “with marvelous foresight” followed the term “anticipated”. Those who borrowed the term "anticipation" from Eddington dropped the qualifiers.

4. For example, see Max Jammer, *Concepts of Space*, (Cambridge: Harvard University Press, 1954), on p.160; A. d'Abro, *The Evolution of Scientific Thought* from Newton to Einstein, (New York: Dover, 1950; Second and enlarged edition of the 1927 original), on p.58; C.W. Kilmister, *General Theory of Relativity*, (New York: Pergamon Press, 1973), on p, 124. Jammer used the term "suggestion" rather than speculation while D'Abro described the concepts as "exceedingly speculative."

5. Jammer, *Concepts*, pp.160-161; D'Abro, *Scientific Thought*, p.58; Banesh Hoffman, *Albert Einstein, Creator and Rebel*, with the collaboration of Helen Dukas, (New York: Viking Press, 1972), on p.176. Jammer used the descriptive terms "fantastic in [Clifford's] own day" rather than "untenable."

6. E.T. Bell, *Men of Mathematics*, (New York: Simon and Schuster, 1965; Reprint of 1937), on p.503.

7. E.T. Bell, *The Development of Mathematics*, (New York: McGraw-Hill, 1945; Second edition of the 1940 original), on p.360.

8. Ruth Farwell and Christopher Knee, "The End of the Absolute: A Nineteenth-Century Contribution to General Relativity," *Studies in the History and Philosophy of Science*, March 1990, 21: 91-121.

9. Joan L. Richards, *Mathematical Visions: The Pursuit of Geometry in Victorian England*, (Boston: Harcourt, Brace, Jovanovitch, 1988).

10. Howard E. Smokler, "W.K. Clifford's Conception of Geometry," *Philosophical Quarterly*, 1966, 16: 249-257; "William Kingdon Clifford," *The Encyclopedia of Philosophy*, edited by Paul Edwards, eight volumes, (New York: Macmillan, 1967), Volume 2: 123-125.

11. James R. Newman, "William Kingdon Clifford," *Scientific American*, February 1953, 188: 78-84, on p.78.

12. E.A. Power, "Exeter's Mathematician - W.K. Clifford, F.R.S. 1845-79," *Advancement of Science*, 1970, 26: 318-328; "The Application of Quantum Electrodynamics to Molecular Forces," a copy was received directly from E.A. Power without references.

13. Gurney, Feza, "Quaternionic and Octonionic Structures in Physics: Episodes in the relation between physics and mathematics," in *Symmetries in Physics*, Proceedings of the First International Meeting on the History of Scientific Ideas held at Sant Feliu de Guixols, Catalonia, Spain, September 20-26, 1983, edited by Manuel G. Doncel, Armin Hermann, Louis Michel and Abraham Pais, (Bellaterra, Spain: Seminari d'Historia de les Cienceis, Universitat Autonoma de Barcelona, 1987): 557-592.

14. J.S.R. Chisholm and A.K. Common, editors, *Clifford Algebras and their Applications in Mathematical Physics*, Proceedings of the NATO and SERC Workshop on Clifford Algebras, Canterbury, UK, September 1985, NATO ASI Series, Series C, Mathematics and Physical Sciences, 183, (Dordrecht: Reidel, 1986).

15. John Archibald Wheeler, *Geometrodynamics*, (New York: Academic Press, 1962). References to Clifford on pp. 8, 123, and 129.

16. Adolph Grünbaum, "The Ontology of the Curvature of Empty Space in the Geometrodynamics of Clifford and Wheeler," in Patrick Suppes, editor, *Space, Time and Geometry*, Synthese Library, (Dordrecht: Reidel, 1973): 268-295, on pp.292-293.

17. Lewis S. Feuer, *Einstein and the Generations of Science*, (New York: Basic Books, 1974), on p.64.

18. Bell, *Men*, p.490.

19. This idea actually goes back to H.J.S. Smith in his introduction to Clifford, *Mathematical Papers*, p.xliii.

20. Henri Poincaré, "Non-Euclidean Geometry," translated by W.J.L., *Nature*, 25 February 1892, 45: 404-407, on p.407; The passage on conventionalism was repeated in *Science and Hypothesis*, translated by W.J. Greenstreet with an introduction by J. Larmor, (New York: Dover, 1952; Reprint of 1905 edition), on p.50.

21. Arthur I. Miller, "The Myth of Gauss' Experiment on the Euclidean Nature of Physical Space," *Isis*, 1972, 63: 345-348, on p.348.

22. For example, see George Chrystal, "Theory of Parallels," the ninth edition, *Encyclopaedia Britannica*, (Edinburgh: Adam & Charles Black, 1885), Volume 18: 254-255; Also see Federigo Enriques, *Problems of Science*, translated by Katherine Royce with a note by Josiah Royce, (London: Open Court, 1914; Original Italian published in 1906), on pp.192-193.

23. Richards, *Visions*, pp. 93-94, 113. Richards uses such terms as "not widely accepted" and "exuberantly speculated" to describe Clifford's physical concepts.

24. Letter from William K. Clifford to Frederick Pollock, partially reprinted in *Lectures and Essays*, edited by Frederick Pollock and Leslie Stephen, two volumes, (London: Macmillan, 1879), Volume I, on p.30. "Solving the Universe" was a phrase often used by Clifford to describe his private thoughts and published work.

25. J.J. Sylvester, "A Plea for the Mathematician," *Nature*, 30 December 1869, 1: 237-239, on p.238; Sylvester's "Plea" was an abridgement of his British Association address, the complete text was printed in the *Reports of the British Association*, (Exeter), 1969: 1-9; The complete text was reprinted with further footnotes in an appendix in J.J. Sylvester, *The Laws of Verse*, (London: Longmans, 1870), and thus reprinted in *Collected Papers*, (Cambridge: at the University Press, 1904): 650-719.

26. The various commentators on "Kant's View of Space" in the pages of the first volume of *Nature* were George Henry Lewes, on pp.289, 334, 386; T.H. Huxley, on p.314; C.M. Ingleby, on pp.314, 361; J.J. Sylvester, on pp.314, 360; G. Croom Robertson, on p.334; and W.H. Stanley Monck, pp.335, 387.

27. William K. Clifford, "On the Aims and Instruments of Scientific Thought," a lecture delivered before members of the British Association at Brighton on 19 August 1872, *Macmillan's Magazine*, October 1872: 499-512; Reprinted in Clifford, *Lectures and Essays*, Volume 1: 124-157.

28. C.M. Ingleby, "The Antinomies of Kant," Nature, 6 February 1873, 7: 262; William K. Clifford, "The unreasonable," *Nature*, 13 February 1873, 7: 282; C.M. Ingleby, "The Unreasonable," *Nature*, 20 February 1873, 7: 302-303.

29. C.M. Ingleby, "Prof. Clifford on Curved Space," *Nature*, 13 February 1873, 7: 282-283, on p.282. Ingleby had already made a general criticism of hyperspace theories in "Transcendent Space," *Nature*, 13 January 1870, 1: 289; and 17 February 1870: 407.

30. Letter from C.M. Ingleby to C.J. Monro, 30 August 1870, #2438, Acc.1063, Monro Collection, Greater London Record Office, London, England. All of the following letters from Ingleby to Monro are from this same collection.

31. Ingleby to Monro, 16 October 1871, #2454.

32. Ingleby to Monro, 29 November 1871, #2469.

33. Ingleby to Monro, 3 February 1872, #2491.

34. Ingleby to Monro, 24 May 1872, #2502B.

35. Letter from C.J. Monro to James Clerk Maxwell, 10 September 1871, Acc.#1063, #2109, the Monro Collection, Greater London Record Office, London, England.

36. James Clerk Maxwell to C.J. Monro, 15 March 1871, partially reprinted in Lewis Campbell and William Garnett, *The Life of James Clerk Maxwel*l, with selections from his correspondence and occasional writings, (London: Macmillan, 1884), on p.290.

37. Monro to Maxwell, 10 September 1871, #2109. Campbell and Garnett only offered a few paragraphs in their publication of this letter and ignored the much larger portion of the letter dealing with the question of a fourth dimension.

38. Note card from James Clerk Maxwell to Peter G. Tait, 11 November 1874, Add. 7655/Ig/72, Maxwell Papers, University Library, Cambridge, England.

39. Letter from James Clerk Maxwell to Peter G. Tait, 9 November 1876, Add.7655/#38, Maxwell Papers, University Library, Cambridge, England.

40. Clifford, *Common Sense*, pp.193-197.

41. Frederick Pollock quoting James C. Maxwell in Clifford, *Lecture and Essays*, p.14.

42. Ingleby to Monro, 9 February 1873, #2540.

43.Clifford, "The unreasonable," p.282.

44. W.K. Clifford, "The Philosophy of the Pure Sciences. II. The Postulates of the Science of Space," *Contemporary Review*, 1874, 25: 360-376, on p.376; Reprinted in *Lectures and Essays*, Volume I, pp.295-323. The lectures on the "Philosophy of Pure Science" were part of an afternoon lecture series at the Royal Institution on 1, 8, and 15 of March 1873. They were subsequently reprinted in *Contemporary Review* and *The*

Nineteenth Century, in October 1874, February 1875, and March 1879, as well as being included in Clifford's Lectures and Essays. 165, 212 and 143 people attended the lectures, respectively, which were substantial crowds for that type of lecture.

45. William K. Clifford, "Preliminary Sketch of Biquaternions," presented on 12 June 1873, *Proceedings of the London Mathematical Society*, 1873: 381-395; Reprinted in *Mathematical Papers*: 181-200.

46. William K. Clifford, "On some Curves of the Fifth Class," and "On a Surface of Zero Curvature and Finite Extent." Only the titles were listed in the *Reports of the British Association*, (Bradford), 1873, 43. No mention is made of these references in any of standard bibliographies on the non-Euclidean geometries. It is nearly as if the lectures were never given.

47. Felix Klein, "Zur nichteuklidische Geometrie," *Mathematische Annalen*, 1890, 37: 544-572. Klein also presented this topic before an American audience in 1893 and published as *The Evanston Colloquium*, *Lectures on Mathematics*, delivered 28 August to 9 September 1893 before members of the Congress of Mathematics held in connection with the World's Fair in Chicago, at Northwestern University, Evanston, Illinois, and reported by Alexander Ziwet, (New York: Macmillan, 1894), on pp.89-90; Wilhelm Karl Joseph Killing, "Clifford-Klein'sche Raumformen," *Clebsch, Mathematische Annalen*, 1891, 39: 257-278.

48. Sir Robert S. Ball quoted by Sir Joseph Larmor in Ball, *Reminiscences and Letters of Sir Robert Ball*, edited by W. Valentine Ball, (London: Cassells, 1915), on p.155; Ball repeats the story in "Non-Euclidean Geometry," *Hermathena*, 1879, 3: 500-541, on p.537.

49. W.K. Clifford, "On the Classification of Loci," read 8 April 1878, *Philosophical Transactions of the Royal Society*, Part II, 1878: 663-681: Reprinted in Mathematical Papers: 305-331; "Applications of Grassmann's Extensive Algebra," *American Journal of Mathematics*, 1878, 1: 350-358; Reprinted in Clifford, Mathematical Papers: 266-276, on p.271.

50. W.K. Clifford, "On the Nature of Things-in-Themselves," *Mind*, 1878, 3: 57-67; Reprinted in *Lectures and Essays*, Volume II, pp.71-88.

51. Peter G. Tait and Balfour Stewart, *The Unseen Universe, or Physical Speculation on a Future State*, (London: Macmillan, 1875). The book was published anonymously at first, but became such an instant success that the authors revealed themselves in later publications. At the time of Clifford's review, the authors were unknown although there was much rumor and speculation as to their identities.

52. William K. Clifford, "The Unseen Universe," *Fortnightly Review*, January-June 1875, 17: 776-793, on p.788; Reprinted in *Lectures and Essays*, Volume I, pp.228-253.

53. William K. Clifford, "Energy and Force," edited by Frederick Pollock and J.F. Moulton, *Nature*, 10 June 1880, 22: 122-124.

54. Peter G. Tait, "Clifford's Dynamic," *Nature*, 23 May 1878, 18: 89-91, on p.91.

55. Unnamed reviewer, "Professor Clifford's Elements of Dynamic," *The Saturday Review*, 22 June 1878, 45: 792-794, on p.793.

56. William K. Clifford, *Elements of Dynamic: An Introduction to the Study of Motion and Rest in Solid and Fluid Bodies*, Part I. *Kinematics*, (London: Macmillan, 1878), on p.221.

57. H.J.S. Smith in W.K. Clifford, *Mathematical Papers*, edited by Robert Tucker, with an introduction by H.J.S. Smith, (New York: Chelsea, 1968; Reproduction of the first edition of 1882).

58. William K. Clifford, *Elements of Dynamic, Book IV and Appendix*, edited by Robert Tucker, (London: Macmillan, 1878), on pp. 59-62.

59. William K. Clifford, "Instruments Illustrating Kinematics, Statics, and Dynamics," in *Mathematical Papers*: 424-440; on p.437. This paper was originally included in the *South Kensington Handbook to the Special Loan Collection of Scientific Apparatus*, 1876, and was also reprinted in Clifford, *Lectures and Essays*, Volume II: 9-30.

60. Richards, *Visions*, pp.143-148, 154; Joan Richards, "Projective Geometry and Mathematical Progress in Mid-Victorian Britain," *Studies in History and Philosophy of Science*, 1986, 17: 297-325, on pp.320-321.

61. Arthur Cayley, Presidential Address to the British Association, *Report of the British Association*, (Southport), 1883: 3-37; Reprinted in *The Collected Mathematical Papers*, thirteen volumes, with volumes I-VII edited by Arthur Cayley, volumes VIII-XIII edited by A.R. Forsyth, (Cambridge: At the University Press, 1889): Volume XI: 429-459, on p.436.

62. Richards, *Pursuit*, pp.90, 138-140.

63. John William Withers, *Euclid's Parallel Postulate: Its Nature, Validity, and Place In Geometrical Systems*, (Chicago: Open Court, 1905), on pp.49-50. This book represented a publication of Wither's doctoral dissertation.

64. William Thomson, Lord Kelvin, "The Wave Theory of Light," a lecture delivered before the Academy of Music, Philadelphia, under the auspices of the Franklin Institute, 29 September 1884, printed in the *Journal of the Franklin Institute*, November 1884, 118: 321-341; Reprinted in Sir William Thomson, *Popular Lectures*, (London: Macmillan, 1888).

65. Letter from Arthur Cayley to Lord Kelvin, 25 March 1889, Add.7342/C63, Kelvin Papers, University Library, Cambridge.

66. Frederick W. Frankland, "On the Simplest Continuous Manifoldness of two Dimensions and Finite Extent," *Transactions of the New Zealand Institute*, 1876, 9: 272-279; And in *Proceedings of the London Mathematical Society*, 1876, 8: 57-64; Also reprinted in Nature, April 1877, 12: 515-517.

67. Simon Newcomb, "Elementary theorems relating to the geometry of a space of three dimensions and of uniform positive curvature in the fourth dimension," *Crelle's Journal fur die reine und angewandte Mathematik*, 1877, 83: 293-299.

68.Felix Klein, "Ueber die sogennante Nicht-Euklidische Geometrie," *Mathematische Annalen*, 1871, 4: 573-611.

69. Robert Stawell Ball, "Measurement," ninth edition, *Encyclopaedia Britannica*, (London: Adam & Charles Black, 1885), Volume 15: 659-668, on pp.664-665; George Chrystal, "Non-Euclidean Geometry," *Proceedings of the Royal Society of Edinburgh*, 1878-1879, 10: 638-664, on p.644; Bertrand Russell, "Non-Euclidean Geometry," in tenth edition, *Encyclopaedia Britannica*, Volume 18: 664-674, on p.668; Bertrand Russell and A.N. Whitehead, "Non-Euclidean Geometry," in the eleventh edition, *Encyclopaedia Britannica*, Volume 11: 724-730, on p.729.

70. Thomas Archer Hirst in William H. Brock and Roy M. MacLeod, editors, *Natural Knowledge in Social Contexts: The Journals of Thomas Archer Hirst*, (London: Mansell, 1980), on p.1828.

71. Simon Newcomb, "Is the Airship Coming?" *McClure's Magazine*, September 1901, 17: 432-435, on pp.432-433.

72. William Skey, "Notes upon Mr. Frankland's paper 'On the Simplest Continuous Manifoldness of Two Dimensions and of Finite Extent'," *Transactions and Proceedings of the New Zealand Institute*, 1880, 13: 100-109.

73. C.J. Monro, "On the Simplest Continuous Manifoldness of Two Dimensions and Finite Extent," *Nature*, 26 April 1877, 17: 547.

74. C.J. Monro, "On the Simplest Continuous Manifold of Two Dimensions and of Finite Extent," *Nature*, 8 July 1880: 218. This letter should not be confused with Monro's earlier note of the same title.

75. C.J. Monro, "Inside Out," *Nature*, 30 May 1878, 18: 115.

76. Simon Newcomb, "Note on a Class of Transformations Which Surfaces May Undergo in Space of More than Three Dimensions," *American Journal of Mathematics*, 1878, 1: 1-4.

77. C.J. Monro, "On Flexure of Spaces," read 13 January 1878, with a comment by Arthur Cayley, *Proceedings of the London Mathematical Society*, November 1877 to November 1878, 9: 171-176.

78. Frederick W. Frankland, *Thoughts on Ultimate Problems: Being a Series of Short Studies on Theological and Metaphysical Subjects*, fifth and revised edition, (London: David Nutt, 1912), on p.13.

79. This meeting was held in Toronto, Canada, on 17 August 1897. Although Frankland's "Theory of Discrete Manifolds" was obscure and copies were difficult to obtain, it was published as Appendix C of *Thoughts on Ultimate Problems*: 37-42.

80. F.N. Cole, "Fourth Summer Meeting of the American Mathematical Society," *Bulletin of the New York Mathematical Society*, October 1897, 4: 1-11, on p.10.

81. Frederick W. Frankland, *Collected Essays and Citations*, Volume I, *Theology and Metaphysics*, 1872-1906, (Foxton, New Zealand: G.T. Beale, 1906).

82. Charles H. Chandler, "Transcendental Space," *Transactions of the Wisconsin Academy of Sciences, Arts and Letters*, 1896-1897, No.11: 237-248, on p.243.

83. George Bruce Halsted, "The Old and the New Geometry," *Educational Review*, 1893, 6: 144-157, on p.150; "Non-Euclidean Geometry," *Popular Astronomy*, 1900, 8: 189-202, on p.189. These are only two of the many examples available.

84. C.S. Peirce in Carolyn Eisele, ed., "The Charles Peirce-Simon Newcomb Correspondence," *Proceedings of the American Philosophical Society*, October 1957, 101: 409-433, on pp.420-423.

85. C.S. Peirce, "The Architecture of Theories," *The Monist*, January 1894, 1: 161-176, on pp.173-174, 176.

86. W.I. Stringham, "Rotation in Four-Dimensional Space," *Johns Hopkins University Circular*, March 1880, 1: 49; "On the Rotation of a Rigid System in Space of Four Dimensions," *Proceedings of the American Association*, 1884, 33: 55-56.

87. Pearson in Clifford, *Common Sense*, pp.lxiii-lxiv.

88. Ingleby to Monro, 15 March 1879, #2649. Ingleby praised Clifford again in another letter to Monro, 19 March 1879, #2650.

89. Frederick Pollock and Leslie Stephen, editors, William K. Clifford, *Lectures and Essays*. These men were also biographers of Clifford. Pollock's touching biography first appeared in *The Fortnightly Review*, 1879, 25: 667-687, and was then revised and published as the introduction to *Lectures and Essays*, Volume 1: 1-43. Leslie Stephen went on to write a short biography of Clifford for the *Dictionary of National Biography,*

from early times to 1900, edited by Sir Leslie Stephen and Sir Sydney Lee, (London: Geoffrey Cumberlege, Publisher to the University, Oxford University Press, 1917), Volume 4: 538-541.

90. Before editing Clifford's *Mathematical Papers*, Robert Tucker had the task of collecting them. He searched far and wide, placing a call for any of Clifford's papers in *Nature*: "Professor Clifford's Mathematical Papers," *Nature*, 26 June 1879, 20: 195.

91. William K. Clifford, *Elements of Dynamic: An Introduction to the Study of Motion and Rest in Solid and Fluid Bodies*, Part 1. *Kinematic, Book IV. And Appendix*, edited by Robert Tucker, (London: Macmillan, 1887).

92. Pearson later repeated these comments in a letter to Ernst Mach, although at that time he claimed to have written them in 1883 which was impossible. The letter has been published by Joachim Thiele, "Karl Pearson, Ernst Mach, John B. Stallo: Briefe aus den Jahren 1897 bis 1904," *Isis*, 1978, 69: 535-542, on p.538.

93. Karl Pearson, "On the Distortion of a Solid Elastic Sphere," *Quarterly Journal of Pure and Applied Mathematics*, 1879, 10: 375- 381; "On the Motion of Spherical and Ellipsoidal Bodies in Fluid Media," *Quarterly Journal of Pure and Applied Mathematics*, 1883, 20: 60-80; Part II: 184-211; "On a certain Atomic Hypothesis," read 2 February 1885, *Transactions of the Cambridge Philosophical Society*, 1883-1889, 14: 71-120, on p.120. The date appearing at the end of the paper was 11 March 1883, the actual date that it had been written; "On a certain Atomic Hypothesis," *Proceedings of the London Mathematical Society*, 8 November 1888, 20: 38-63.

94. Karl Pearson, "Note on Twists in an Infinite Elastic Solid," *Messenger of Mathematics*, 1883, 13: 79-95; "On the Generalised Equations of Elasticity, and their Application to the Wave Theory of Light," *Proceedings of the London Mathematical Society*, 1889, 20: 297-350.

95. Karl Pearson, "Ether Squirts. Being an attempt to specialize the form of ether Motion which forms an Atom in a Theory propounded in former papers," *American Journal of Mathematics*, 1891, 13: 309-362.

96. Pearson, "Ether Squirts," p.309. However, Pearson gave a slightly more extensive explanation on pp.312-313.

97. Letter from Karl Pearson to Robert J. Parker, dated 6-4-85, file #922, Pearson Papers, University College, London.

> Cambridge very quiet, did nothing but look over proofs & talk mathematics, metaphysics with Macauley. He does not see how I can create the universe out of empty space, by twisting it, and is a perfect slave to the matter superstition. So Sir William Thomson, who has been writing

> about the weight of the ether, as if empty space could weigh anything! I am going to weigh a twist! ... That might mean something."

98. Pearson in Clifford, *Common Sense*, p.203.

99. Charles T. Whitmell, *Space and Its Dimensions*, (Cardiff: South Wales Printing, 1892), on p.24.

100. Frankland, *Thoughts*, pp.12-13.

101. Karl Pearson, "Matter and Soul," read before the Sunday Lecture Society on 6 December 1885, and printed as a pamphlet by the Society; Reprinted in *The Ethic of Freethought and other Addresses and Essays*, second edition, (London: Adam and Charles Black, 1901; Originally printed in 1887): 21-44, on p.30-32.

102. Pearson, "Matter," pp.28-29.

103. Pearson, "Matter," pp.29-30.

104. Karl Pearson, Review of Elements of Dynamic - Part 1. Kinematic, Book IV, and Appendix, *The Athenaeum*, 16 July 1887, No.3116: 86-87, on p.86.

105. Karl Pearson, *The Grammar of Science*, (London: Walter Scott, 1892; Third edition, London: Adam and Charles Black, 1911; New York: Meridian Books, 1961; Reprint of the third edition of 1911), on p.229.

106. Pearson, *Grammar*, p.229.

107. Letters from Lord Kelvin to Karl Pearson, 10 January 1893, 16 January 1893, 2 March 1893, 11 March 1893, file #871/1, Pearson Papers, University College, London.

108. Letter from Karl Pearson to Lord Kelvin, undated, #104, Pearson Papers, University College, London. This letter is listed as an essay on "The Nature of Physical Space" in the catalogue to the Pearson Papers, rather than a letter to Kelvin. It is probably from the period about 1892-1893, the time during which Pearson was arguing with Kelvin over the nature of the hypothetical ether.

109. Robert S. Ball, "On the Small Oscillations of a Rigid Body about a Fixed Point under the Action of any Forces, and, more particularly, when Gravity is the only Force acting," read 24 January 1870, *Transactions of the Royal Irish Academy*, 1860-1870, 24: 593-627; A summary was published in *Proceedings of the Royal Irish Academy*, 1870-1874, 5, second series: 11-14.

110. Arthur Buchheim, "A Memoir on Biquaternions," *American Journal of Mathematics*, 1884, 7: 293-326; "On Clifford's Theory of Graphs," *Proceedings of the London Mathematical Society*, 12 November 1885, 17: 80-106; "On the Theory of

Screws in Elliptic Space," *Proceedings of the London Mathematical Society*, 10 January 1884, 15: 83-98; 13 November 1884, 16: 15-27; 10 June 1886, 17: 240-254; 11 November 1886, 18: 88-96.

111. Sir Robert S. Ball, "The Twelfth and Concluding Memoir on the 'Theory of Screws,' with a Summary of the Twelve Memoirs," read 8 November 1897, *Transactions of the Royal Irish Academy*, 1896-1902, 31: 145-196, on p.536.

112. Joseph Larmor in Ball, *Reminiscences*, pp.154-155.

113. Robert Stawell Ball, "Non-Euclidean Geometry," *Hermathena*, 1879, 3: 500-541; "Notes on Non-Euclidean Geometry," *Reports of the British Association*, 1880, 50: 476-477; "On the Elucidation of a Question in Kinematics by the Aid of Non-Euclidean Space," *Reports of the British Association*, (York), 1881, 51: 535-536; "Certain Problems in the Dynamics of a Rigid System Moving in Elliptic Space," read 14 November 1881, *Transactions of the Royal Irish Academy*, 1880-1886, 28: 159-184; "Notes on the Kinematics and Dynamics of a Rigid System in Elliptic Space," read 9 June 1884, *Proceedings of the Royal Irish Academy*, 1884-1888, 4: 252- 258; "Note on the Character of the Linear Transformation which Corresponds to the Displacement of a Rigid System in Elliptical Space," read 9 November 1885, *Proceedings of the Royal Irish Academy*, 1884-1888, 4: 532-537; *Dynamics and the Modern Geometry: A New Chapter in the Theory of Screws*, the Cunningham Memoir, No.IV., (Dublin: Published by the Academy at the Academy House, 1887).

114. Robert Stawell Ball, "The Distance of Stars," read 11 February 1881, *Proceedings of the Royal Institution of Great Britain*, 1879-1882, 9: 514-519, on p.519.

115.Ball,"Measurement," p.664.

116. Robert Stawell Ball, "On the Theory of Content," read 12 December 1887, *Transactions of the Royal Irish Academy*, 1889, 29: 123-182.

117. Ball, "Content," p.151.

118. Olaus Henrici, "The Theory of Screws," *Nature*, 5 January 1890, 42: 127-132, on p.131.

119. Robert Stawell Ball, "A Dynamical Parable," *Nature*, 1 September 1887, 36: 424-429; Originally presented as the presidential address to the Physical Section of the British Association, and published in the *Reports of the British Association*, (Manchester), 1887, 57; Reprinted in Ball, *Treatise*: 496-509, on pp.508-509.

120. Robert Stawell Ball, *A Treatise on the Theory of Screws*, (Cambridge: Cambridge University Press), on p.519.

121. Charles H. Hinton, "What is the Fourth Dimension?" originally published in the *Dublin University Magazine*, 1880, and then republished in the *Cheltenham Ladies' College Magazine*. The publisher printed it as a separate pamphlet in 1884 with the subtitle “Ghosts Explained” added.

122. Hinton's "What is the Fourth Dimension?" was first published as a pamphlet before it was added to a collection his other essays and published in *Scientific Romances*, two volumes, (London: Swann & Sonnenschein, 1884-1886), Volume 1: 1-32; Reprinted in *Speculations of the Fourth Dimension: Selected Writings of Charles H. Hinton*, edited by Rudolf v.B. Rucker, (New York: Dover, 1980): 1-22.

123. Hinton, "What?" in *Speculations*, pp.16-20.

124. Hinton, "A Plane World," in *Speculations*: 23-40, on pp.36- 37. Hinton also developed the idea of "twists" as a physical concept, independent of electrical phenomena, in "Many Dimensions," in *Speculations*: 67-79, on pp.74-75. Both of these essays were published in *Scientific Romances*.

125. Hinton, "A Picture of Our Universe," in *Speculations*, on pp.52-55. Originally published in *Scientific Romances*.

126. Rucker in Hinton, *Speculations*, p.v.

127. Simon Newcomb, "On the Fundamental Concepts of Physics," presented before the Washington Philosophical Society, roughly typed copy in the Simon Newcomb Papers, Box #94, Library of Congress; Abstract printed in *Bulletin of the Washington Philosophical Society*, 1888/1891, 11: 514-515.

128. William Walter Rouse Ball, "A Hypothesis Relating to the Nature of the Ether and Gravity," *Messenger of Mathematics*, 1891, 21: 20-24.

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133. Karl Pearson, "Modern Physical Ideas," Chapter X in *Grammar of Science*, Part I, *Physical*, third edition, (London: Adam & Charles Black, 1911): 355-387.

134. R.S. Heath, "On the Dynamics of a Rigid Body," *Philosophical Transactions of the Royal Society of London*, 1884, 175: 281-324.

135. Homersham Cox, "Homogeneous Coordinates in Imaginary Geometry and their Applications to Systems of Forces," *Quarterly Journal of Mathematics*, 1881, 18: 178-215; "On the Application of Grassmann's Ausdehnungslehre to different kinds of Uniform Space," read 20 February 1882, *Transactions of the Cambridge Philosophical Society*, 1882, 13, Part II: 69-143.

136. Cayley, "Presidential Address," pp.435-436.

137. Samuel Roberts, "Remarks on Mathematical Terminology, and the Philosophic Bearing of Recent Mathematical Speculations concerning the Realities of Space," *Proceedings of the London Mathematical Society*, 9 November 1882, 14: 5-15, on p.9.

138. Johann Bernhard Stallo, *The Concepts and Theories of Modern Physics*, edited by Percy W. Bridgman, (Cambridge, Massachusetts: The Belknap Press of Harvard University, 1960; Reprint of the second edition of 1884; First edition published in 1881), on pp.222-279, especially pp.225-228, 244 and 251.

139. Klein did not tackle the problem of Clifford's model of a non-Euclidean space until 1890. It is strange that Klein was supposed to have returned to a more physical conception of mathematics in the 1890's with his work on the theory of the top, at least according to the standard historical view. However, given Clifford's physical emphasis on geometry, it would be reasonable to conclude that Klein's rehabilitation of this geometrical model which Clifford developed in 1873 might have initiated Klein's new found interest in the physical aspects of geometry before his development of the theory of the top. Klein presented his theory of the top before an American audience in 1896. *The Mathematical Theory of the Top*, lectures delivered on the occasion of the *Sesquicentennial Celebration of Princeton University*, 12-15 October 1896, (New York: Scribners' Sons, 1897).

140. Poincare, "Non-Euclidean," p.406.

141. Edward Kasner, "The Present Problems of Geometry," in Harry J. Rogers, editor, *Congress of Arts and Sciences, Universal Exposition, St. Louis, 1904*, Volume I, *Philosophy and Mathematics*, (Boston: Houghton & Mifflin, 1905): 559-586, on p.559; Reprinted in *Bulletin of the American Mathematical Society*, 1905, 11: 283-314, on p.559.

142. Kasner, "Present," p.562.

143. Corrado Segre, "On Some Tendencies in Geometric Investigations," *Bulletin of the American Mathematical Society*, 1904, 11: 442-468, on pp.446-447, or 462.

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