Twist 'til we tear the house down!

By **James E. Beichler**

PART I

Nearly a half century before Einstein developed his general theory of relativity, the Cambridge geometer William Kingdon Clifford announced that matter might be nothing more than small hills of space curvature and matter in motion no more than variations in that curvature. These ideas were further elaborated in Clifford's *Common Sense of the Exact Sciences* of 1885,[1] partially written and edited by Karl Pearson six years after Clifford's unfortunate death from consumption.

The short abstract of 1870 in which Clifford explained his model of space, "On the Space-Theory of Matter,"[2] has long been recognized in studies on general relativity and its history, but Clifford's concepts of space and their relationship to physics have been limited to the role of "anticipation"[3] of Einstein's theory. Within this context, Clifford's model has been branded a "speculation"[4] that was "untenable"[5] during his brief professional career. E.T. Bell has gone so far as to liken Clifford's "brief prophecy"[6] to hitting "the side of a barn at forty yards with a charge of buckshot."[7]

Recently, there has been some renewed interest in the relationship between Clifford's theory and its role in the development of modern physics. Ruth Farwell and Christopher Knee have looked at Clifford's work as a "nineteenth century contribution to general relativity."[8] Joan Richards has written on the development of non-Euclidean geometry in Victorian England, a movement in which Clifford played a significant role,[9] while Howard E. Smokler, a philosopher, has taken a new look at Clifford's concepts within a philosophical context.[10]

This apparent renewal of interest is not a totally new phenomenon, but has been occurring in regular cycles for some time. Nearly forty years ago, James R. Newman noted that the "neglect of Clifford [was] difficult to explain,"[11] yet nothing strikingly new has been published on Clifford. The newest revelations offered by these scholars in recent years have not yet shed the preconceived historical outlook on Clifford that can be found in older post-relativity publications. They offer more of the same old inaccuracies tempered by a few bright spots of original historical research.

The renewed interest extends beyond the historical significance of Clifford's work to his mathematical system of biquaternions, first developed in 1873. A.E. Power, a mathematician at the University College in London, has published articles comparing Clifford's mathematical system to modern work in both quantum mechanics and relativity.[12] Feza Gurney has written of the "hope and disappointment connected with the role of quaternions in physics,"[13] another episode in which Clifford's mathematical system can be found to play an important role. A conference dedicated to Clifford's mathematical system and its application to modern physical theory was held at Canterbury, England, in 1985.[14] The interest of physicists in Clifford's mathematical work received a large boost some years ago when John Archibald Wheeler developed some of Clifford's ideas in his "Geometrodynamics,"[15] but that approach to relativity theory seems to have been abandoned. Adolph Grünbaum has written something of a philosophical obituary for it.[16]

All of these new inquiries into Clifford's work are inadequate from the historical perspective. In many respects, they merely perpetuate the small myths concerning Clifford's role in the development and popularization of non-Euclidean geometry and physical space. In turn, these myths form part of a legend that has grown up around Einstein's discovery of general relativity and its acceptance by the scientific community. Einstein's discovery has been presented as the first successful attempt to describe physical space as non-Euclidean.[17] This statement is beyond debate, but past historical studies on the subject either imply or state that attempts to associate the new forms of geometry with the physical world before Einstein were either non-existent or quite rare and isolated, i.e., "untenable." This attitude forms the general historical context against which Clifford's accomplishments are normally evaluated.

Regardless of the recent interest in Clifford's work, no one has noted the logical paradoxes that arise from these commonly accepted historical views. While it is generally accepted that Clifford's ideas were "untenable" in the 1870's, no one has yet addressed the complementary issue of how scientific attitudes had changed so much in the intervening years that nearly the same concept was "tenable" during Einstein's early career. In fact, general relativity was accepted quite rapidly by scientists, philosophers, mathematicians and scholars, as well as educated laymen, in spite of the radical notion of representing matter by space curvature.

On the other hand, if it could be demonstrated that the non-Euclidean geometries were popular enough that Einstein's version of curved space was not as radical in 1915 as Clifford's in 1870, then it would become necessary to find the event or factor between the two periods when the turning point in attitude occurred. The explanation of the rapid acceptance of relativity would be made easier, but at the expense of a cherished legend. The point at which the shift from a purely mathematical non-Euclidean geometry to the belief in a possible physical interpretation of such geometries could not be so easily pinpointed. The obvious question would then become, what was Clifford's connection to this evolutionary process of accepting a physical non-Euclidean space? Historians could take the Whiggish way out of the dilemma and state that Einstein's concept was "tenable" because it was correct and/or accepted, but history cannot be served by having it both ways by denying the affect of Clifford's work while accepting Einstein's theory as "tenable" in the presence of a void.

Those authors who characterize Clifford's work as "speculation," support their conclusions by further stating that he had no followers who continued his work after his death,[18] or he never published his theory.[19] Since Clifford's was the strongest Victorian voice supporting a physical interpretation of non-Euclidean geometry, these accusations and misrepresentations imply that the physical interpretations of the non-Euclidean geometries were disregarded by the vast majority of scholars before Einstein.

In this regard, Poincare's conventionalist attitude, that it is preferable to change the laws of physics (or optics) and retain Euclidean space rather than consider the possibility that space might be non-Euclidean,[20] is usually cited as representing the prevalent view of the scientific and academic communities immediately prior to relativity. By accepting Poincare's view as scientific doctrine during that period, historians and others have neglected the fact that Poincare's conventionalism was a reaction to the growing tendency to associate the non-Euclidean geometries with physical space well before the end of the nineteenth century. It was geometric escapism. The story that has evolved around these misinterpretations of the historical record forms one of the basic foundations upon which Einstein's theory is considered a solid break with past theoretical work in physics and attitudes on the non-Euclidean geometries.

Several years ago, Arthur I. Miller attempted to destroy another of the pillars of mathematical history that might challenge Einstein's absolute originality in relating a non-Euclidean geometry to physical space. He argued that J.K.F. Gauss' survey measurements from three mountaintops in Hanover in the 1820's originally had nothing to do with space curvature as many authors have indicated. According to Miller, that particular interpretation of Gauss' survey was made only after the development of general relativity.[21] However, there is strong evidence that Gauss' survey measurements were interpreted as a measure of space curvature long before relativity theory was first introduced.[22] This evidence also emphasizes the fact that the physical consequences of non-Euclidean geometry were a popular subject for discussion and debate within the scientific and academic communities during the late nineteenth century. The popularity of the non-Euclidean geometries and the possibility that they rendered physical consequences that could be investigated implies that there is no real basis for accepting the conclusion that Clifford's ideas were "untenable." Further investigation of Clifford's work would seem warranted, but has not yet been carried out. While admitting the necessity of investigating Clifford's work, recent authors have perpetuated the mistaken images of the past.[23]

These apparent paradoxes can be dispensed with quite readily by taking a fresh look at both Clifford's work and its reception, but this must be done within the greater historical context of the attitudes toward the non-Euclidean geometries during the period before relativity theory. The story that emerges from such a study demonstrates that Clifford's ideas were both "tenable" and popular, and raised profound questions within the academic community on the role of non-Euclidean geometry in physics. Clifford was working on a specific theory, which was partially completed before his untimely death. Therefore, his ideas were not "speculations," but a serious effort to "solve the universe,"[24] as Clifford would say. He also had followers who attempted to extend his work after his death.

Enough of Clifford's theory can be reconstructed from his various papers to indicate the general principles upon which he based his theory. In essence, Clifford's theory cannot be evaluated from just a cursory reading of his "Space-Theory" and the *Common Sense*, as past writers and investigators have done. All of his published papers must be investigated to understand the depth and breadth of his theoretical outlook. It was in this manner that Clifford's colleagues and peers interpreted his concepts. However, Clifford's mathematical theory was so abstract and so intimately bound to quaternions that it had minimal affect on the later development of relativity and perhaps disguised Clifford's work from the scrutiny of later scholars and historians. The purely mathematical portion of Clifford's theory was continued by Sir Robert S. Ball and Arthur Buchheim, among others, and primarily involved the mechanics of motion in an elliptical space.

The fundamental element of space curvature in Clifford's mathematical model was the twist, which he hoped to use to describe electromagnetic and atomic phenomena. Karl Pearson continued Clifford's development of the twist without reference to its relation to space curvature in his own development of the "ether-twist." Pearson's as well as these other extensions of Clifford's work were all bound to the Victorian principles and attitudes toward science that were already in decline before relativity struck. Correctly or incorrectly, they suffered from either an association with quaternion algebras or ether-vortex theories, or both, at the time when these physical concepts lost favor within the scientific community.

Of greater historical importance was the more general, philosophical concept of space as expressed by Clifford and its relation to the mathematical studies of non-Euclidean geometry. Included within this perspective would be the general model of non-Euclidean space presented in Clifford's "Space-Theory" abstract, but not exclusively by that presentation. In this regard, Karl Pearson, Frederick W. Frankland and Charles H. Hinton spread Clifford's ideas. Clifford's purely mathematical studies were not immune from involvement in this aspect of the non-Euclidean debate, but the complexity of the mathematics involved allowed only the best and brightest mathematicians to draw conclusions regarding the physics of space and time directly from Clifford's mathematics. On the other hand, the philosophical contributions allowed anyone who could read and use their imagination to draw conclusions from Clifford's more general concepts.

Two generations of thinkers who expressed an interest in the non-Euclidean geometries and hyperspaces separated Clifford's original work from general relativity. During that time, the popularity of physical interpretations of the non-Euclidean geometries grew. When Einstein first completed and published his theory, it found an eager audience, which already accepted the possibility that physical phenomena could be affected by space curvature. Although many scholars and laymen added their own thoughts to this general attitude during the decades before the adoption of relativity theory, Clifford's contributions went beyond all others in both content and breadth of view. His "Space-Theory" set the limits to which all others attained in their belief of a physical non-Euclidean space before Einstein institutionalized the concept that matter could be represented by space curvature.

II. Clifford's Theory and the Issue of "Tenability"

The first public announcement that Clifford had been working on a new concept of space and matter came in J.J. Sylvester's presidential address before the British Association in 1869. The speech was subsequently published with footnotes in *Nature*, where it became available to a much larger audience with a more varied background. What they first learned of Clifford's work was that our space of three dimensions might be "undergoing in a space of four dimensions ... a distortion analogous to the rumpling" of a piece of paper.[25] Sylvester also committed himself to a belief in a fourth dimension and mentioned that Immanuel Kant thought of space as a "Form of Intuition." Sylvester's interpretation of Kant's doctrine on space immediately triggered a debate over the Kantian meaning of the phrase "Form of Intuition" and Kant's notion of space as "a priori."[26] The ensuing debate over Kant's concept of space thus became the first line of defense for those who accepted the absolute truth of a three-dimensional Euclidean space.

As the initial stages of this debate began to subside, C.M. Ingleby, a contributor to the Kant debate, took up the crusade against Clifford's concepts. Both the early Kant debate and the row over Clifford's statements took place in the "Letters to the Editor" column of *Nature*, where all could follow. Ingleby was known as an expert on Shakespeare, but he was also well acquainted with other areas of philosophy, especially Kant's work. Ingleby first criticized Clifford's characterization of Kant's concepts as expressed in Clifford's address, "On the Aims and Instruments of Science,"[27] presented before the British Association in 1872.

The infraction against Kant was small and after the initial round of charge and countercharge[28] Ingleby progressed to the real point of contention. Clifford had presented his first statements on the general concept of space and non-Euclidean geometry in this speech and had put a scare into an unnamed friend of Ingleby's.[29] It had not been Clifford's inconsequential comments on Kant that had raised Ingleby's ire, but the challenge to Kant's notions of space as implied by Clifford's references to the non-Euclidean geometries.

For historical purposes, this minor debate between Clifford and Ingleby might seem of little significance, but it was just the tip of the iceberg in a larger debate proceeding out of public view within the scientific community. Ingleby was an intimate friend of Sylvester and they discussed the physical consequences of a four-dimensional non-Euclidean space in private. Evidence of these discussions comes from yet another source, the surviving portions of an ongoing correspondence between Ingleby and C.J. Monro.

Ingleby and Monro were debating the concept of a possible higher dimension to space as early as August of 1870.[30] In October of 1871 Ingleby informed Monro that Sylvester maintained "there is nothing in geometry that is not wholly based on order in time."[31] The following month, Ingleby stated that Sylvester had reached a "new position, that the mathematicians are now in possession of evidence that space is curved. This is now what he says and sticks to."[32] In the next letter, Monro was once again updated on Sylvester's opinions of curved space and informed that Sylvester had "assured [Ingleby], ..., that he had reason to believe that our space is curved. But [Sylvester] did not base this on Celestial Observations."[33] Unfortunately, Sylvester would not tell Ingleby why he thought space was curved. If he had, historians would have a record of Sylvester's thoughts on the subject today.

Thus, before the late winter or early spring of 1872, Ingleby had only related what he thought were Sylvester's private thoughts and concerns of a curved space, as best he could, to Monro. Although the thoughts were attributed to Sylvester, Sylvester had informed Ingleby that the "mathematicians" accepted these notions. Then, in a letter written on 24 May 1872, Ingleby spoke of "the speculations of Riemann, Helmholtz and Clifford."[34] From that day forward, whenever he spoke of a curved space, the reference was to Clifford, not Sylvester. Clifford's opinion on the subject had been elevated to a position above that of Sylvester's, and it was implied that Sylvester was following Clifford's lead on the subject of curved space. When Sylvester referred to "the mathematicians" he had really been referring to those who followed Clifford's ideas. Clifford's time had come, and the issue of curved space became an open argument between Clifford and Ingleby in the pages of *Nature*.

Monro was also a friend and correspondent of James Clerk Maxwell, who obtained for Monro a membership in the London Mathematical Society[35] as well as a friend of Arthur Cayley. In March of 1871, Maxwell wrote to Monro and enumerated his arguments against a fourth dimension.[36] Another letter, this time from Monro to Maxwell in September of 1871, indicates that these thoughts were part of an ongoing discussion between Maxwell and Monro.[37]

Maxwell may not have been completely convinced by his own arguments against the non-Euclidean geometries and hyperspaces. He was still somewhat perplexed on the issue and not totally sure of his own concepts on 11 November 1874 when he wrote to his friend and fellow physicist Peter G. Tait, once again expressing his opinions.

> The Riemannshe Idee is not mine. But the aim of the space-crumplers is to make its curvature uniform everywhere, that is over the whole of space whether that whole is more or less than infinity. The direction of the curvature is not related to one of the x y z more than another or to -x -y -z so that as far as I understand we are once more on a pathless sea, starless, windless and poleless totus feres abque rotundus.[38]

Maxwell was somewhat incredulous and ambivalent toward the new concept of a curved space, but none-the-less concerned. His reference to the "space-crumplers" indicates his disagreement with them, but also indicates that he could not ignore their arguments. Nor could he ignore the possibility that the non-Euclidean hypothesis might bear some relevance to physics.

The depth of the debate within the scientific community is demonstrated by the fact that Maxwell's opinion in this instance came in answer to a question posed by Tait in a letter to Maxwell just two days earlier. Both men were concerned with the new mathematical hypotheses, but Tait seemed slightly more willing to accept their possibility. Tait exhorted Maxwell to "Xplane why it is bosh to say that the Riemannsche Idee may, if it is found to be true, give us <u>absolute</u> determinations of position."[39] It is obvious that Tait attributed some special knowledge of Riemann's geometry to Maxwell or that he knew Maxwell was in contact with those who did have such knowledge. Otherwise, he would not have made this inquiry. It is equally obvious that both Maxwell and Tait were searching for counter-arguments to the "space-crumplers" continuing onslaught.

The term "space-crumplers" referred directly to Clifford with regard to his stated opinions on the feasibility of using curved space to develop a physical theory of "solving the universe." The idea that absolute position could be found via the use of space curvature was a basic tenet of Clifford's geometrical model of space, as later expressed in *Common Sense*.[40] Maxwell did not agree with these concepts, but he did know of them and still had a great deal of admiration for Clifford and Clifford's abilities. He offered a glowing letter of recommendation when Clifford applied for a professorship at University College in London.[41]

Ingleby also admired Clifford even while he debated with him. His venomous attack on Clifford in *Nature* was not as serious as it would seem. Ingleby wrote to Monro that Monro would not agree with some of his criticisms of Clifford. Indeed, Ingleby himself saw "things in it to be excepted to. But [he had] an object to serve thereby." Ingleby could not "seem ignorant of such a speculation" as Clifford announced in his open portrayal of non-Euclidean space[42] and he felt obliged to overreact in public to the implications of Clifford's geometrical concepts of space. He only wished to "open the oyster" and air his arguments against the new ideas espoused by Clifford in a public forum. Thus, in private Monro tempered his attack on Clifford, but did not alter his position.

Clifford refused to further answer Ingleby's charges within the pages of *Nature*. Instead, he promised Ingleby that his answers would be forthcoming in a series of lectures at the Royal Institution.[43] The lectures to which Clifford referred were given in March of 1873. The three lectures constitute Clifford's "Philosophy of the Pure Sciences." They answered all of Ingleby's charges within a far more comprehensive philosophy of science. The second lecture of the series, "The Postulates of the Science of Space," dealt specifically with Clifford's concept of space. This lecture became one of Clifford's most popular expositions of the non-Euclidean geometries as well as his general concept of space. Clifford ended the lecture with a statement that he often found relief from the boredom of our homaloidal space by picturing an elliptic space which he hoped would someday explain physical phenomena.[44]

The year of 1873 marked a distinct turning point in Clifford's quest for a space-theory of matter. In the early summer, Clifford published the essay "Preliminary Sketch on Biquaternions,"[45] which described a new calculus of twists and screws. This calculus was the three-dimensional counterpart of an elliptic space. Clifford accomplished this feat by combining concepts from William Rowan Hamilton's quaternions and some of the features of Hermann Grassmann's "Ausdehnungslehre." It seemed that Clifford's work was reaching a point of climax, given his recent public lectures and his new mathematical system. Then, in the British Association meeting of 1873, Clifford offered a paper entitled "On some Curves of Zero Curvature and Finite Extent."[46] In this paper, Clifford presented a new non-Euclidean geometry that exhibited Euclidean flatness over large distances, but Riemannian characteristics in the infinitesimal connections between consecutive points of space. This geometry was an extension of his algebra of biquaternions.

Both systems, the new geometry and biquaternions, made use of Clifford's concept of geometric parallelism whereby parallel lines need not exist in the same plane. It is curious that Clifford never published an explanation of this geometry while his mathematical theory of space and matter suffered the same fate. It is quite possible, given Clifford's basic tenet that geometry is a physical science, that this geometric model in fact represented his spatial model of matter and that he so stated in his presentation. However, as far as history is concerned, that conjecture will probably never be proven. What history has recorded is that Clifford's new geometry would have died away had not Felix Klein and W. Killing revived and begun to develop Clifford's new geometry about 1890.[47]

At this meeting of the British Association, Clifford also met with Robert Stawell Ball and Klein. In long, all night discussions, he converted Ball to the non-Euclidean point of view and traded ideas on the non-Euclidean geometries with Klein.[48] Clifford had known of Ball's work on screws before this meeting and had adopted the screw system for his own use in the system of biquaternions. In so doing, he explored the geometry of motion to a far greater extent than Ball would during his own lifetime.

From this time forward, Clifford's emphasis changed from his general model of space and matter to the dynamical study of matter in motion in an elliptical space via his use of biquaterions, screws and twists. It can be assumed that the scientific community did not receive Clifford's new geometry very well. Otherwise, there would have been more development of it before Klein revived it in 1890 and Clifford would have continued to develop it before his unfortunate death. Perhaps Clifford was discouraged by this lack of faith in his geometry. Clifford could not have known that his new geometry was far too advanced for a scientific community that was just beginning to cope with the repercussions pursuant to the discovery of ordinary non-Euclidean geometries. The more immediate concern of science was the development and understanding of Maxwell's theory of electromagnetism, and it was to that end that Clifford applied his biquaternions and twists.

It is not that Clifford ignored the more general problems of space. He published papers in 1878 related to general spaces, "On the Classification of Loci" and "Applications of Grassmann's Extensive Algebra."[49] In both cases, Clifford dealt with the problem whereby properties in a flat space of lesser dimensions were analogous to properties in an elliptic space of a higher dimension by one. The connection of this research to his physical theory should be obvious. He was looking for the mathematical terminology to convert physical properties in a three- dimensional space to a curved four-dimensional space.

Clifford also penned several related articles on philosophical subjects during this period. In the essay "On the Nature of Things-in-Themselves,"[50] Clifford introduced the concept of "mind-stuff" which offered a philosophical method to deal with the problem that the human mind could not perceive curved space. He also wrote a critique of the *Unseen Universe*[51] in which he implied that the ether was secondary to space curvature.[52] Clifford was clearly trying to establish a complete philosophical picture of the universe, rather than just a space-theory of matter.

At this point, Clifford's thoughts on "solving the universe" evolved in several directions at once, but his true passion was a description of matter in motion. To that end, he published what would prove to be his magnum opus, the *Elements of Dynamic*, in 1878. The *Elements* was both simple enough for anyone with a mathematical education through trigonometry to understand as well as deep enough for anyone with experience in advanced mathematical physics to find enlightening. Clifford was trying to rebuild the mathematical universe on the basis of non-Euclidean principles without appearing to do so. In this first volume of the *Elements*, Clifford combined quaternion algebra, projective geometry and vector techniques to describe kinematical motion, but he was insidiously preparing his readers for the acceptance of the biquaternions, which would turn his work into the mathematical equivalent of a non-Euclidean space. This fact can only be understood by a comparison to Clifford's other published work from the same period of time. It was within this context that his colleagues understood his work.

Although the title to the book promised a study of dynamics, Clifford only delivered kinematics. In his grand scheme of an ultimate reality there was no such thing as a force, therefore a study of the dynamics of motion was unnecessary. Clifford only believed in energy of motion or kinetic, and energy of relative position or potential, but not in force. He had explained this in a lecture in 1872, but the lecture was never published in its entirety. An abridged version of "Energy and Force" was published in *Nature* from Frederick Pollock's notes of the lecture after Clifford's death.[53] This reduction of force to energy as a property of space fits Clifford's overall scheme of reducing three-dimensional dynamics to the four-dimensional kinematics in an elliptic space and conforms with his "Space-Theory" model.

In the *Elements*, Clifford refused to mention quaternions, projective geometry and such advanced terminology and symbols as found in those studies. In their place he introduced many visually descriptive terms such as twists, squirts, sinks, shells, vortices and so on. Tait praised Clifford's use of quaternion methods in his review of the book for *Nature*, but condemned Clifford's introduction of such terms as confounding the issue.[54] On the other hand, Clifford was praised by others for using such terms so that even the most modestly educated person could understand his explanations.[55]

The concluding statement of the book more accurately described what Clifford was trying to accomplish. Clifford refrained from use of the term ether throughout the book. In only one case of an example did he stray from this pattern. Otherwise, all references to an elastic medium, or in this one particular case an infinite body, could just as easily be interpreted as representing the whole of curved space.

> Thus we have shown that if the expansion and the spin are known at every point, the whole motion can be determined, and the result is, that every continuous motion of an infinite body can be built up of squirts and vortices.[56]

Since the "squirts" and "vortices" were in essence composed of twists of "stuff" within an infinitely extended elastic "stuff," Clifford's system of algebra could be of instrumental use in describing such motions. On one level the "stuff" could be interpreted as the ether, on another level space curvature, and on yet another level Clifford's "mind-stuff." Thus, the system of kinematics which Clifford proposed in the *Elements* was a thinly disguised application of his "Space-Theory of Matter," and the *Elements* was a literary vehicle for the development and application of his biquaternion system of algebra.

The relation of the *Elements* to Clifford's "Space-Theory" is not at all obvious to anyone unfamiliar with Clifford's concept of space and complete mathematical researches. It would appear to be just another Victorian attempt to explain ether vortices, but it was much more than that in actuality. For this reason, historians, philosophers, scientists and other scholars who later studied Clifford's published writings have utterly failed to recognize the import and extent of Clifford's theoretical researches.

The second volume of the *Elements* was unfinished at Clifford's death. The fragments that remained were collected and published by Robert Tucker,[57] but they did not measure up to the promise offered by the first volume. Some hints were given in this reconstruction of Clifford's book of the direction in which Clifford hoped to take physical theory. For example, he related gravitation to a strain in space,[58] a fact confirmed by an earlier publication,[59] but he did not delve into this matter any further than a few simple statements. He also planned to give a more precise definition of matter than just "stuff" whose mass was determined by comparison to some agreed upon standard, but this definition was never completed. Clifford stressed the mathematics of screws which would imply that he planned to use screws and biquaternions for most of his mathematical analysis of physics.

The model of space upon which Clifford settled could be briefly described as a four-dimensional elliptic space in the large. The constant of curvature was too small for detection through astronomical observations, but that fact did not negate the possibility that space could be other than Euclidean. Our three-dimensional space was probably considered a boundary between two four-dimensional sections of space. It may not be proper to use the term space in the sense of four dimensions, since there is evidence that Clifford considered the fourth dimension to be time. However, it is more likely that Clifford believed in a purely four-dimensional space with time as a separate but connected quality or quantity. A four-dimensional kinematics would be adequate to describe all physical phenomena, but at the same time it would be analogous to a three dimensional dynamics. Clifford's own geometry could circumvent the problem of non-observation of space curvature on the astronomical scale since his geometry approximated Euclidean space in the large.

The infinitesimal scale of nature presented other problems. On this scale the connections of contiguous points of space exhibited curvature in the fourth dimension. The three-dimensional analogue of this curvature was an elastic medium in which twists were the most fundamental element. The twists, in turn, composed vortices and squirts that supplied strains in the elastic medium, which gave rise to electromagnetic and gravitational forces. This particular reconstruction of a model of space conforms well to all aspects of Clifford's work, but it must be remembered that it is a reconstruction. Several parts of this model are confirmed by the later work and comments of Clifford's students and followers. It is far too tempting to attempt such a reconstruction even though its historical accuracy is debatable. What is assured is that some model, at least similar to this one, represented the goal toward which Clifford was moving.

The chief mathematical obstacle to Clifford's theory was the projective interpretation of space. Clifford freely used projective methods in his mathematical researches and his work has been categorized as projective.[60] However, the projective view of space implies an intrinsic alteration in the definition of distance while strictly adhering to a three-dimensional Euclidean space of experience rather than adopting an extrinsic curvature of space. Although Clifford used projective methods, he was not a dedicated projective geometer in the same sense as Arthur Cayley. On the whole, Clifford's work was not projective.

Cayley argued for the projective view at least as late as 1883 in his address before the British Association.[61] Modern historians[62] and Victorian scholars[63] alike have accepted this view as an accurate description of Cayley's attitudes toward physical space. Indeed, this was the face he put on for the public, but in private he was unsure of his own position in the face of Clifford's arguments.

In 1889, ten years after Clifford's death, a group of Lord Kelvin's popular lectures were published.[64] Upon receipt of his copy of the book, Cayley questioned Kelvin's use of vortices of ether to describe physical phenomena. In a letter to Kelvin, Cayley wrote that

> In the lecture on the wave theory, you parenthetically ignore the notion of the curvature of space - Clifford would say that, going far enough, you might come - not to an end - but to the point at which you started. I have never been able to see whether this does or does not assume a four-dimensional space as the locus-in-quo of your [vortical] & therefore finite space.[65]

Cayley, possibly the staunchest advocate of Euclidean three-dimensionality, even while a friend, teacher and colleague of Clifford, had been swayed by Clifford's arguments. This admission demonstrated a crack in the thick veneer shrouding both Cayley's and the Victorian dedication to Euclideanism since Cayley was the inventor of a mathematical system, the projective geometry, which offered the only logical alternative to the radical concept of a curved space. Here we see the steadfast pillar of Victorian geometry with cracks heretofore unnoticed by historians.

Under these circumstances, the use of words such as "speculation," "prophecy" and "untenable" to describe Clifford's work and its reception among his peers is, to say the least, historically inaccurate as well as unfounded. Clifford had begun to publish his theory while the extent of his research far exceeds anything implied by either of the terms "speculation" or "prophecy." The term "untenable" implies nearly absolute rejection of Clifford's theory and concepts, the case of which has been demonstrated as an historical inaccuracy. The new historical issue thus becomes a question of whether Clifford's research died with Clifford. If Clifford's ideas and their influence on the study of non-Euclidean geometry ended with his death, as other authors have contended, then Clifford's work could have had no influence on general relativity. However, Clifford had followers who continued different facets of his research and further popularized his concepts.

PART II

ENDNOTES

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Top of Page

YGGDRASIL Homepage

Go to Issue 2