III: Discussion and Criticism

Kaluza was able to derive both the trajectories of charged particles and uncharged particles within the framework of the coherent space-time structure of his model. This was done in a simple and straightforward manner. As has been shown, his theory resulted in parametric representations in five-dimensional space which coincide with families of geodesics, each of which depends on different values of the ratio e/m. Previously, this result could not have been obtained in the Riemannian space of General Relativity, but had to be carried out in different Finsler spaces, dependent on the different values of e/m. So, Kaluza's theory was a success in what it attempted to accomplish: the unification of the electromagnetic and gravitational fields. However, the success of Kaluza's theory has been greatly diminished by some serious criticisms of the theory. Due to these criticisms, the theory has not been generally adopted and is still looked upon by many with disfavor.

All criticisms of Kaluza's theory deal either directly or indirectly with Kaluza's basic assumption of a fifth dimension. Many physicists consider a formalism such as the one used in the theory as artificial since the universe as sensed is four-dimensional. The artificiality appears since Kaluza's five-dimensional assumption is presented only as a mathematical formalism and the fifth coordinate is totally devoid of any physical content. "The success of a language adopted to a five-dimensional manifold is, ..., only a way of concealing the lack of developments truly adaptable to the four-dimensional universe, which remains the true physical universe."[67] This question of the 'reality' of the fifth dimension is quite crucial to any theory based on a fundamental assumption of a five-dimensional component. Kaluza left the question entirely open, but other theorists since have sought to clarify this oversight.

Klein, Kaluza's most immediate successor, tried to use the five-dimensional hypothesis to account for quantum effects, as have others after Klein, while some scientists sought to extend or change the five-dimensional space structure in other ways to meet the challenge of criticism regarding the reality of the fifth dimension. Such theories attempted to explain why only four dimensions are physically discernible; the projective theories explained away the fifth dimension geometrically, while the Einstein-Bergmann, Einstein-Bergmann-Bargmann and the Jordan-Thiry theories were attempts to give some physical significance and content to the fifth dimension. Einstein's final comment on this question can be found in the second appendix to the fourth edition of *The Meaning of Relativity*. Here he stated that any such theory can be regarded if and only if (my strong qualification, not Einstein's) it could be shown why all empirical data leads to a strictly four-dimensional universe. So, the five-dimensional theories cannot be forsaken due to our failure to either detect or sense its existence, as long as scientists can justify their first assumption of a fifth dimension in the light of phenomena in our world.

On the other hand, the question of the mathematical role that the fifth dimension plays in our universe is intimately related to the question of its reality and existence, once the fifth dimension has been assumed. This role can be considered in several different although interrelated ways. Since we have no intuitive or "pretheoretic account of even the qualitative features of a possible fifth dimension,"[68] we have no guidelines by which to consider the fifth-dimensional component, leaving the role that the fifth dimension plays within the theory unclear and open to speculation. This gives very wide latitude to any possible theoretical application, perhaps too wide to justify such theories in the long run.[69]

A crucial factor in our normal space-time is that of the 3+1 division of space-time (or its mathematical signature of +,+,+,-). By giving the variable γ_{00} a positive value rather than a negative one, so that bodies always attract each other (Pauli demonstrated that the positive factor is related to the gravitational constant and thus attractive in nature),[70] the fifth dimension is spacelike rather than timelike.[71] The choice of a negative value would have given the fifth dimension timelike qualities. This choice of sign seems to be a matter of mathematical expediency in the absence of sound intuitive judgment. A present lack of physical evidence of the fifth dimension actually precludes the question of whether it is ultimately spacelike or timelike. Kaluza's theory would seem to indicate the spacelike nature of the fifth dimension, however, the case is not yet closed regarding this factor. It should be remembered "Even if it is in some way spacelike, the fifth dimension differs much more from the three ordinary spacelike dimensions than does time. We would need additional conceptual distinctions, besides that of spacelike versus timelike, to separate it from the other four."[72]

In like manner, the role played by the condition of cylindricity of the fifth-dimensional component has a mathematical basis in the absence of intuitive guidelines. The cylindrical condition allows the four dimensions of space-time to be independent of the fifth dimension, which in a way serves to explain why there is no physical evidence of a fifth dimension. Since all observables in our world are four-dimensional, they seem to be independent of a fifth dimension. Mathematically speaking, the cylindrical condition limits the kinds of coordinate transformations possible, allowing only those which lead to covariant field equations.[73] This means that the fifth coordinate must play a special role in our physical world.

This special role is evident under the cut-transformation, where the symmetrical derivatives of the A-curve vanish while the anti-symmetrical derivatives of A_μ remain allowing a correlation with the electromagnetic field. Thus the cylindrical condition is necessary to the successful unification of the electromagnetic and gravitational fields. Yet, the imposition of the cylindrical condition has been an important point of criticism of the earlier five-dimensional theories and thus constitutes a major weak point in these theories. A special status or peculiarity of the fifth component of the field is revealed in this condition, thus the condition has been interpreted by some as being too restrictive or merely an 'additional condition' which is unnecessary. The cylindrical condition is used to limit the fifteen possible field equations to the fourteen necessary to describe the electromagnetic and gravitational fields.

It has been argued that a condition less stringent than the cylindrical condition could be used yielding the same results for the fourteen equations describing electromagnetism and gravitation, while leaving the fifteenth equation intact to describe other field phenomena. This operation would leave the five-dimensional theory more general in its approach. "The condition thus makes it impossible to achieve a complete synthesis in the way that, for example, Maxwell's theory achieved a synthesis of the electric and magnetic fields."[74] This weakness has lead to attempts to change or modify the cylindrical condition. In the projective theories, the cylindrical condition is interpreted quite naturally as a projective condition that shows the purely auxiliary role of the five-dimensional space.[75]

The cylindrical condition can also be said to lead to a mere codification within the five-dimensional formalism, such that it is a mathematical convenience rather than a physical characteristic of space.[75] In that case, it has been assumed that the fifth dimension is not real as well as being cylindrical. But, if the five-dimensional space is assumed to be real, such that there is a true five-dimensional geometry or space structure, rather than a geometrical (mathematical only) formalism representing space-time, then the cylindrical condition can not be modified or dropped altogether. This approach to the problem was taken by Einstein and Bergmann, extended by Einstein, Bergmann and Bargmann and finally by Podolanski. In their theories, the extra dimension (or dimensions in the case of Podolanski's six-dimensional theory) is considered to be real, but of special structure. Instead of a cylindrical condition, the Einstein-Bergmann (and its later version with Bargmann's collaboration) theory allows for a fifth dimension which is closed with respect to the four dimensions of space-time, whereas Podolanski's six-dimensional theory has a "laminated structure such that all points in a given layer correspond to a single point in the four-dimensional space-time."[77] These theories are then able in some manner to answer those criticisms that attack the fifth dimension's dependence on a cylindrical condition.

As stated above, the five-dimensional theories seem to be merely formal (in that they are mathematical formalisms regardless of the reality of the fifth dimension), non-intuitive (in that we have no previous experience of a five-dimensional space-time structure) and ad-hoc.[78] These theories also stand accused of being a mere synthesis and ultimately non-predictive,[79] in that Kaluza's theory does not expand upon the Einstein-Maxwell equations. It has also been argued that the Kaluza theory, in its design and conditions, only reproduces the Einstein-Maxwell equations within the formalism of the combined field structure that it builds, while various interpretations can be made so that it matches the Einstein-Maxwell equations exactly. Some interpretations, which may not have been scientifically justified, except through hindsight, were made to exactly correlate the Kaluza theory with the Einstein-Maxwell theory. Two instances of this 'hindsight' can be found in the identification of the scalar quantity $\alpha\beta^2/2$ in the final equation of Kaluza's theory with the gravitational constant k in the General Theory of Relativity and the identification of the field vectors A_m ($= \gamma_{0i}$) with the electromagnetic potentials ϕ_i, simply because they share the same mathematical characteristics.

There are no sound scientific or physical justifications for either of these identifications. But nor are there any scientific or physical reasons why they cannot be made. So, it has been further argued by some critics that electromagnetism has not been incorporated into the field structure in a natural way[80] as was originally desired. By identifying the A_m with the electromagnetic potential ϕ_i, no geometrical character has actually been established for the electromagnetic field. Nor has it been demonstrated "that only Maxwell's equations could be combined with Einstein's into a single formalism."[81] Pauli went so far as to say that Kaluza's representation "is in no way a 'unification' of the electromagnetic and gravitational fields. On the contrary, every theory which is generally covariant and gauge-invariant can also be formulated in Kaluza's form."[82] In other words, it has been claimed that the Kaluza theory does not render a single field from which both electromagnetism and gravitation can be "exclusively" derived.

The search for a unified field theory originated in the dissatisfaction that some scientists experienced with the field equation defining space structure into which the electromagnetic tensor was somewhat artificially introduced. Kaluza's theory allows for the geodesics of charged particles to be found within the combined field of electromagnetism and gravitation, a fact that should have overcome the dissatisfactory nature of the field. However, the field equations of the combined field are derived from a variational principle, in which the Lagrangian still appears as the sum of two terms,

$$= Fg(R + \frac{k}{2} F_{ik} F^{ik}) \ .^{83}$$

Here, R is the scalar representing space curvature and F_{ik} (or F^{ik}) is the electromagnetic field strength. Even though the field has been unified into a single structure, there is still no single tensor that can be used to represent both the electromagnetic and gravitational aspects of the field. The addition of the electromagnetic tensor seems to have been transferred from the field equations of General Relativity to the Lagrangian of the five-dimensional theory. So the theory still seemed synthetic to scientists without completely establishing the geometric character of either the electromagnetic field or the totally combined field. In this sense, the original purpose of the search for a unified field theory has still not been completely satisfied, at least according to those who voiced this particular criticism. These facts do not mean that either the Kaluza or the Kaluza-Klein theories have reached a dead end. The theory still points to further possibilities that have not yet been fully investigated.

IV: Conclusion

The mere assumption of a five-dimensional space-time structure can take two basic forms, both of which are not as independent of the other as one would expect or hope. First of all, the five-dimensional space-time may be used as a mathematical formalism alone, which has no more meaning than to allow for the extra variables needed to incorporate the electromagnetic field into a single space-time structure. In this manner, the fifth dimension is assumed to have no physical meaning. It was this form that Kaluza's theory assumed. In the second type of these theories, the fifth dimension is given an actual physical reality. It is a basic necessity of this type of theory to explain why the fifth dimension seems to be beyond the natural experience of human perception.

The assumption of a fifth dimension in either of the forms above gives rise to a number of criticisms. It seems that the different criticisms would be specific to either a theory that claims to be 'only a mathematical formalism' or one which claims 'to represent a physically real space-time structure,' not both. But this is not completely true! On the other hand, the Kaluza theory is criticized for mathematical assumptions that serve to reproduce the necessary equations (Maxwell's and Einstein's) without any physical basis. One such aspect of Kaluza's formalism is the assumption of a positive one value for γ_{00}. This value is critically associated with the condition of cylindricity. Yet, even though the Kaluza theory is only meant to be a mathematical formalism, the value of +1 and the condition of cylindricity are both criticized for being assumptions which cannot be correlated with physical reality, even while five-dimensional theories in general are criticized by way of the fact that we have no "pretheoretic" or intuitive ideas of how to conceptualize a fifth dimension.

t seems as though there is no difference between these two forms of theories as far as the criticisms are concerned. Once a fifth dimension is assumed in any way, means, shape or form, the theory is not allowed to remain only mathematical, but it is required to be physically real as well. In this sense, if the fifth dimension is introduced merely as a mathematical tool and meets with any success in the scientific community, then new questions must be answered. Why isn't our normally sensed space-time five-dimensional as is the successful mathematical theory? Of course, if the five-dimensional space-time would supply additional information, solve more problems or otherwise be of operational advantage with respect to a four-dimensional space-time, it would still be justifiable in spite of the reality question. On the other hand, if real space-time is expected to be five-dimensional, why then isn't the fifth dimension sensible? It seems that the mere mention of a fifth dimension in any manner immediately raises profound philosophical questions as well as scientific questions regarding the reality of a fifth dimension. This fact is recognized indirectly in Einstein's statement that "any" such theory must account for the "apparent" restriction of our experienced world to only four dimensions of the continuum. This introduces a maddening circle of independence for the various criticisms of these types of theories as well as both forms of the five-dimensional assumptions. Therefore, there is only one criticism in the end, and that is the criticism of the need to adopt a fifth dimension itself.

The main criticisms are of the variety that the five-dimensional theories are artificial, synthetic and so on. This is a gross understatement of the success and simplicity of the Kaluza theory in deriving Maxwell and Einstein's equations as well as the geodesics for charged particles in the combined field. The fact that the theory "in the least duplicates" the accepted results of Maxwell and Einstein, should indicate to scientists the very real possibilities of using five-dimensional space-times in physics. The Kaluza theory and the extensions that it spawned, while being criticized for their admitted shortcomings, should not be seen as an endpoint and cast aside. In spite of their limited success, they should offer a starting point for a more successful view of nature. If, then, by providing a theory which answers the criticisms while retaining the success of Kaluza's theory, a new overall worldview should emerge and a change in the present paradigm of physics would seem to be a matter of serious speculation. The fact that extensions of Kaluza's theory have met with little support in the scientific community (a fact which is true for other unified field theories such as the non-symmetric field) would seem to attest to both the weakness of these theories as well as the strength of the present paradigm of science (the quantum mechanical approach), but not to any ultimate change or alteration in that paradigm.

Perhaps, then, the method of pursuing extensions of Kaluza's theory is the wrong approach to the problem. If the only real criticism to these theories is a criticism regarding the adoption of the fifth dimension itself, then more emphasis should be placed on the philosophical implications of a real fifth dimension. The mathematics could follow later, rather than their leading directly to a non-intuitive, non-philosophically sound mathematical artifice, as has been done in the past. A direct assumption of a real fifth dimension could be made as the basis of a new theory. Given this assumption, a new philosophy could be built utilizing qualitative speculations regarding how this five-dimensional model might account for simple mechanical and electromagnetic phenomena.

Questions regarding natural phenomena could be answered along this line of reasoning in order to build a "pretheoretic" philosophy for the five-dimensional space. Any five-dimensional theory should be able to answer several questions in modern physics. For example, how does Lorentz contraction "look" in a five-dimensional space-time continuum? How can a five-dimensional space-time account for a mass increase with an increase in relative velocity? How can electromagnetic fields fit into a five-dimensional scheme of space-time structure? Why (or how) does the four-dimensional space-time continuum appear to curve in the five-dimensional worldview? These and similar questions may not be considered valid inquiries for scientists at first glance, but given the fact that humans seem to have no intuitive judgment concerning the fifth dimension, there is no reason why such questions cannot be asked and answered.

No one seems to have yet developed such a philosophy, or rather; I have had difficulty in discovering anyone who has published anything regarding such a philosophy. At least there have been no references made to one. Even so, it has been recognized that such a philosophy is needed. Peter Bergmann has stated, "The potentialities of hyper-dimensional unified field theories seem great. ... At present, however, all such theories - as unified field theories generally - lack a convincing and complete physical interpretation. In this way they are little worse off than Einstein's theory of gravitation itself. But unlike the theory of gravitation they are also marred by the absence of a compelling logical necessity," or of a compelling operational necessity. Bergmann made these comments during the Summer Institute of Theoretical Physics at Brandeis University in 1957, well before the modern use of the Kaluza-Klein theory (since about 1980) as a basis of a 'theory of everything.'

If a philosophy (in the form of a conceptual background) could be developed, whereby either a 'pretheoretic' view and/or an intuitive sense of a five-dimensional space-time was made available for use to either supplement the existing theory or form the basis of a new theory, then science could only benefit from the effort. The main problem with a field theory of this type lay in its inability to act as a basis for physical predictions. A conceptual view of the fifth dimension would more than likely lead to a predictive role for such a theory, in such a way that predictions made by the theory could be used to substantiate both the theory and the five-dimensional hypothesis upon which it is based. One important area where this could possibly occur is in the realm of the quantum. By unifying gravitation and electromagnetism, while incorporating the quantum, Bergmann's prerequisite of a "compelling logical necessity" would be fulfilled.

Such theories have already been sought without a conceptual background. These theories took the easier path by either explaining away the reality of the fifth dimension or ignoring its reality altogether. But the fact that the five-dimensional hypothesis has been consistently, if only sporadically, applied to the problem of unification, would tend to indicate that similar theories based upon philosophical and conceptual considerations are feasible. Without a conceptual foundation to act as a guide in the development of these theories, they will always be susceptible to the same criticisms as stated above. There is ample room within science for the development of five-dimensional unified field theories, but whether the necessity of this hypothesis is present is open to question. In the words of James Perlman, "In spite of the handicaps of physical reality or of the complexity of the basic assumptions (at least at this point), any five-dimensional theory would still be justified if it can be shown to have, or at least to promise additional information to solve additional problems, to operationally simplify, to relate or unify seemingly separate principles, or to have other operational or theoretical benefits."[84] So the search for a conceptual philosophy of the fifth dimension is thus a legitimate endeavor for science.

NOTES

1. Herman Weyl, *Space, Time, Matter*. (New York: Dover, 1952; Reprint of English translation of *Raum, Zeit, Materie* of 1918)

2. Theodor Kaluza, "Zum Unitätsproblem der Physik," *Sitzungsberichte der Preussische Akademie der Wissenschaft*, (1921): 966-972.

3. Weyl noted that Riemann's geometry only went "halfway towards attaining the ideal of a pure infinitesimal geometry," (Weyl, *Space*, p.102) so he introduced a gauge system into the space-time geometry. In his physical model, the parallel transfer of a length in the field would allow a change in the basic unit of length according to the gauge. This change showed the presence of distant curvature and allowed for the introduction of electromagnetism into the metric of the four-dimensional space-time curvature. This initial attempt by Weyl was shown to yield physical consequences that were contrary to experimental evidence and thus proved a failure. However, Eddington, Einstein and others, later extended his work with affinely connected spaces, or those spaces governed by parallel displacement of a vector.

4. In the 1940's, Erwin Schrödinger returned to the work started by Weyl, Eddington and Einstein, based on the affine connection of the space-time curvature, to derive a new unified field theory. In 1923, Eddington made the assumption that the affinity was symmetric in the lower indices of the gamma function in general relativity, such that $\Gamma^{k}_{ml} = \Gamma^{k}_{lm}$. This allowed the sixty-four possible gamma functions to be reduced to forty. But Schrödinger went still further and considered the case where this symmetry was discarded allowing for the derivation of field equations which very nearly matched those derived by Einstein (at about the same time) by yet another method.

5. Einstein's last attempts to derive a unified field theory began about 1945. These were based upon the fact that the metric tensor g_{ik} could be made to conform to a Hermitean symmetry whereby $g_{ik} = g_{ki}$. The metric tensor could then be split into symmetric and anti-symmetric parts. The anti-symmetric portion of the metric tensor was finally equated to the electromagnetic field while the symmetric portion remained related to the gravitational field. The various versions of this theory appeared as appendices in the successive editions of Einstein's book *The Meaning of Relativity*, with the final version appearing in the final edition of 1955, the year of Einstein's death. Einstein realized that the theory was still incomplete, so he outlined a program whereby others could complete the theory after his death.

6. Reprinted in Varadaraja V. Raman, "Theodor Kaluza," *Dictionary of Scientific Biography*, edited by Charles Gillespie. (New York: Charles Scribner and Sons, 1974), Volume VII: 211-212. Einstein encouraged Kaluza to pursue such an approach, submitting that this was an entirely original point of view.

7. Albert Einstein, *The Meaning of Relativity*, sixth edition. (Princeton: Princeton University Press, 1956): 93-94. The contradiction stems from the fact that Einstein seems to change from time to time in his support for five-dimensional theories. It could be concluded that his scientific training and intuition taught him to be leery of so radical a concept as this, so he never really supported it fully while still considering it a truly original idea. According to Professor James Perlman (in a private communication), "Einstein preferred, other things being equal, simplicity in assumptions in his earlier years, but appreciated the rigorous scholarship of the five-dimensional attempts of others at that time. As he came into snags at later times with the four-dimensional models, he paid more personal attention to the five-dimensional possibilities."

8. See note #4. Eddington did not make any substantial contributions to the search for a unified field theory after this attempt. However, he did continue his own independent theoretical work, which ended two decades later with his Fundamental Theory.

9. Albert Einstein, "Kaluza's Theory of the Correlation of Gravitation and Electricity: Parts I & II," *Sitzungsberichte der Preussische Akademie der Wissenschaft, Berlin*, 6 (1927): 23-25, 26-30.

10. Louis DeBroglie, "L'Univers A Cinq Dimensions et La Mécanique Ondulatoire," *Le Journal de Physique et le Radium*, 8 (Fevrier 1927): 65-73.

11. Wolfgang Pauli, *The Theory of Relativity*. (New York: Pergamon Press, 1958): 230: "Kaluza and Klein derived, however, a further interesting result. They computed the scalar P ..." and so on. This implies some degree of collaboration between Kaluza and Klein.

12. Kaluza's works in bibliographical form are found in Poggendorf, VIIA, Pt.2 (1958): 684. I have not seen these, so he may have done more work on his theory for publication, but there are no references to any

other work.

13. James E. Beichler, "Twist 'til we tear the house down," *Yggdrasil: The Journal of Paraphysics*, 1 (Winter Solstice 1996), http://members.aol.com/Mysphyt1/yggdrasil-2/contents.htm.

14. Oskar Klein, "Quantentheorie und Fünfdimensionale Relativitätstheorie," *Zeitschrift fur Physik*, (1926): 895-906; also "The Atomicity of Electricity as a Quantum Theory Law," *Nature*, 118 (9 October 1926): 516; *And "Zur Fünfdimensionale Darstellung der Relativitätstheorie," Zeitschrift fur Physik*, 46 (1927): 188-208.

15. Oswald Veblen and Banesh Hoffman, "Projective Relativity," *Physical Review*, 36 (1 September 1930): 810-822.

16. Wolfgang Pauli, "Über die Formulerung der Naturgesetze mit fünf homogenen Koordinäten," *Annalen der Physik*, 18 (1933): 305-366.

17. D. Van Dantzig and J.A. Schouten, "Four-Dimensional Interpretation of the Unified Field Theory," *K. Akad. Amsterdam*, *Proc*., 34 (1931): 1398-1407.

18. Albert Einstein and W. Mayer, "*Einheitliche Theorie von Gravitation und Electrizität," Sitzunberichte der Preussische Akademie der Wissenschaft, Berlin*, 25 (1931): 541-557; 11-12 (1932): 130-137.

19. Albert Einstein and Peter G. Bergmann, "On a Generalization of Kaluza's Theory of Electricity," *Annals of Mathematics*, 39 (July 1938): 683-701.

20. Albert Einstein, Peter Bergmann and V. Bargmann, "On the Five Dimensional Representation of Gravitation and Electricity," in the *Theodor von Karman Anniversary Volume*, (Pasadena, 1941): 212-225.

21. Pasqual Jordan, "Erweitung der projectiven Relativitätstheorie," *Annalen der Physik*, 6 (1947): 219.

22. Y. Thiry, "Les Équations de La Théorie Unitaire de Kaluza," *Comptes Rendus*, 226 (1948): 216-218; "Étude Mathématique des Équations d'une Théorie Unitaire à quinze Variables de Champs," *Comptes Rendus*, 226 (1948): 1881-1882.

23. J. Podolanski, "Unified Field Theory in Six Dimensions," *Proceedings of the Royal Society*, 201 (1950): 234-260.

24. By 'mainline' theories I mean those theories that other scientists normally cited in their own articles and research on the subject.

25. A great deal of work has been done on extending the Kaluza-Klein theory since about 1980. Present theories hypothesize a ten dimensional space-time continuum to account for physical reality as it appears to us. It is a believed that these ten dimensions (or in some cases more) were equally represented at the time of the big bang, but six of the ten were diminished to infinitesimally small proportions at the expense of the four dimensions that we now experience in our world. The leading proponent of these theories is Michio Kaku. An account of these theories is given in Kaku's book *Hyperspace: A Scientific Odyssey through Parallel Universes, Time Warps, and the 10th Dimension*. (New York: Oxford University Press, 1994).

26. H.T. Flint, "Relativity and the Quantum Theory," *Royal Society, Proceedings*, 117 (1 February 1928): 630-637; "The Quantization of Space and Time," *Physical Review*, 74 (15 July 1948): 209-210,

27. D. Meksyn, "Unified Field Theory, Part I: Electromagnetic Field," *Philosophical Magazine*, 17 (1934): 99-113; "Unified Field Theory, Part II: Gravitation," *Philosophical Magazine*, 17 (February 1934): 476-482.

28. G. Vranceanu, "La Théorie Unitaire des Champs et les Hypersurfaces Non-holonomes," *Comptes Rendus*, 200 (17 June 1935): 2056-2058; "Non-Holonomic Unified Field Theory," *Journal de Physique et le Radium*, 7 (December 1936): 514-526.

29. KentanoYano, "Unitary Non-Holonomous Field Theory, Parts I and II," *Phys. Math. Soc. Japan, Proc*., 19 (October 1937): 867-896; (November 1937): 945-976; and "Théorie de la Relativité," *Comptes Rendus*, 204 (1 February 1937): 332-334.

30. H.C. Corben, "A Classical Theory of Electromagnetism and Gravitation. I. Special Relativity," *Physical Review*, 69 (1 and 15 March 1946): 225-234; "Special Relativistic Field Theories in Five Dimensions," *Physical Review*, 70 (1 and 15 Dec 1946): 947-953.

31. D.K. Sen, *Fields and/or Particles.* (New York: Academic Press, 1968): 77. In this equation $R_{\mu\lambda}$ is the contracted Christoffel tensor, R is the scalar representing space curvature, k is the gravitational constant and $T_{\mu\lambda}$ is the tensor representing matter.

32. Sen, 85. The standard notation $\{^{\beta}{}_{\lambda\mu}\}$ represents the Christoffel symbol of the second kind where $\{^{\beta}_{\lambda\mu}\} = \frac{1}{2} g^{\beta\nu}(\delta g_{\lambda\nu}/\delta x^{\mu} + \delta g_{\mu\nu}/\delta x^{\lambda} - \delta g_{\lambda\mu}/\delta x^{\nu})$.

33. Marie-Antoinette Tonnelat, *Einstein's Unified Field Theory*, translated by Richard Akerib. (New York: Gordon & Breach, 1966): 5. The metric of the Finsler space is equal to the Riemannian metric plus an added term which depends on the e/m ratio and the electric potential ϕ_{μ}.

34. The most general Riemannian metric has far more than 10 terms, but the restrictions in the general theory of relativity reduce the metric to 10 independent terms for gravitation alone. This sentence refers to the special metric of gravitation.

35. John C. Graves, *The Conceptual Foundations of Contemporary Relativity Theory*. (Cambridge: M.I.T. Press, 1971): 206.

36. Pauli, 227-228.

37. Peter Bergmann, *An Introduction to the Theory of Relativity*. (New York: Dover Press, 1976): 254.

38. Einstein and Bergmann, 683-4.

39. Banesh Hoffman, *Einstein: Creator and Rebel*. (New York: Viking Press, 1974): 403.

40. Graves, 257.

41. Pauli, 228. Here the subscript '5' refers to the fifth-dimensional component in deference to the quotation from Pauli. For the rest of the paper the subscript '0' will refer to the fifth-dimensional component. Other authors use one or the other. Also, Roman subscripts normally refer to the indices 1,2,3,4 for normal space-time and Greek subscripts 0,1,2,3,4 for five-dimensional space-time.

42. 'A' was used earlier in this paper to refer to the vector magnetic potential as is commonly done. However, when using A in terms of A-lines, A-curves and A-cylindricity, the A should not be assumed to have anything to do with the vector magnetic potential.

43. Einstein and Bergmann, 684.

44. The operator ';' commonly refers to a differentiation process.

45. Einstein and Bergmann, 684.

46. The quantity $\Gamma^{\nu}{}_{\alpha\beta}$ is a coefficient of connection and equals $\{^{\nu}{}_{\alpha\beta}\}$.

47. Einstein and Bergmann, 685.

48. Graves, 256.

49. Pauli, 230.

50. Graves, 256.

51. Pauli, 228.

52. Winston E. Kock, "Unified Field Theories," *Current Science*, No.11 (May 1938): 546-548, on p.547. The prime simply means the coordinates are given in the transformed system.

53. Einstein and Bergmann, 687.

54. Einstein and Bergmann, 686.

55. Einstein and Bergmann, 687.

56. Kock, 548.

57. In this case $\gamma_{ik}-\gamma_{0i}\gamma_{0k}=\gamma_{\mu\nu}-\gamma_{0\mu}\gamma_{0\nu}/\gamma_{00}$ *where* $\phi_i=\gamma_{0\mu}/\gamma_{00}$.

58. Pauli, 228-229.

59. Bergmann, 265.

60. Kock, 548.

61. Pauli, 229.

62. Bergmann, 267.

63. Bergmann, 268.

64. Sen, 87.

65. Sen, 87 or Jagdish Mehra, *Einstein, Hilbert and the Theory of Gravitation*. (Dordrecht, Holland: D. Reidel, 1973): 53.

66. Bergmann, 254.

67. Marie Antoinette Tonnelat, *The Principles of Electromagnetic Theory and of Relativity*, translated by A.J. Knodel. (Dordrecht, Holland: D. Reidel, 1966): 403-404.

68. Graves, 258.

69. Since there are no perceptual guidelines to the concept of the fifth dimension, scientists enjoy and employ a great deal of latitude in their speculations on its physical characteristics. However, the variety of these speculations is far too wide. Science must find some specific physically related phenomena to prove the reality of a fifth dimension. This task is complicated by the fact that those scientists who employ the five-dimensional hypothesis cannot come to any agreement as to the characteristics of the fifth dimension.

70. Pauli, 230.

71. Graves, 258.

72. Graves, 258.

73. Tonnelat, *Unified Field Theory*, 7.

74. Tonnelat, *Principles*, 404.

75. Tonnelat, *Unified Field Theory*, 7.

76. Tonnelat, *Unified Field Theory*, 8.

77. Tonnelat, *Principles*, 403.

78. Graves, 257.

79. Since the theory only duplicates the Einstein-Maxwell equations and adds no new knowledge it is said to be non-predictive. In other words, it can predict nothing new beyond what can already be found with the previously known Einstein-Maxwell theory. If it added something new and was thus predictive, it would be possible to test and verify the five-dimensional hypothesis used.

80. Graves, 257-258.

81. Graves, 257-258.

82. Pauli, 230.

83. Pauli, 230.

84. In a private communication from Professor James Perlman at San Francisco State University in 1980.

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