Editor's Introduction to

The Fourth Dimension

By

C. H. Hinton

Charles Howard Hinton began writing popular descriptions of a possible fourth dimension of physical space in the 1880's, just as he graduated from Oxford University in England as a geometer. His descriptions are extensions of the mathematical models of W.K. Clifford, whose theories regarding four-dimensional spaces were quite popular. At first, Hinton was interested in realizing or mentally picturing the fourth dimension, but later gave up that task as impossible. Nor did he attempt to do more than describe the properties of the fourth dimension. He did not try to build a comprehensive mathematical model of the fourth dimension even though he described the possible physical attributes that could be expected from a four-dimensional perspective.

He later moved to America and eventually began work at the Naval Observatory, probably due to the influence of Simon Newcomb who had retired as director of that institution. It was at this time that Hinton finally began to develop his mathematical model more rigorously, again probably due to Newcomb's influence. He presented a more complete model before the Washington Philosophical Society in 1902. The present paper, as published in *Harper's Magazine* (Volume 109, June 1904: 229-233), represents Hinton's popular description of that final theoretical work. It is unknown whether Hinton extended this model any further before his death several years later.

Hinton's theoretical work also came a few years before Einstein's special theory of relativity was developed, after which a successful four-dimensional space-time became known to science. But Hinton's model represents a purely four-dimensional space, independent of time, as did the other four-dimensional models that were popular during the last decades of the nineteenth century.

Like the other historical papers offered in this edition of the *Yggdrasil*, the work must be read within the context of Hinton's era of time and the scientific and cultural norms under which it was authored. Quite simply, scientific norms have changed since this paper was written, but the paper still has intrinsic value. Hinton offers three cases that would prove the existence of a real physical fourth spatial dimension. Case 1 deals with a specific molecular structure that has since been explained by science without resorting to a four-dimensional space. In case 3, Hinton tries to explain a particular instance of electrical induction that is also well understood by present science without resorting to a higher-dimensional space. So these cases are irrelevant by today's standards for demonstrating the existence of a fourth dimension of space. However, Hinton's last case, number 2, is not altogether negated by today's standards. Although his particular examples of right and left handedness is no longer valid, there are indeed unexplained instances of right and left-handedness in nature, such as the spin of elementary particles, to which his example could be applied. Whatever the case may be, Hinton's final assessment that we can only regard a four-dimensional space as possible if three-dimensional mechanics fails to explain known physical phenomena still rings true today. However, today we are dealing with a five-dimensional space-time instead of a four-dimensional space with a separate dimension of time

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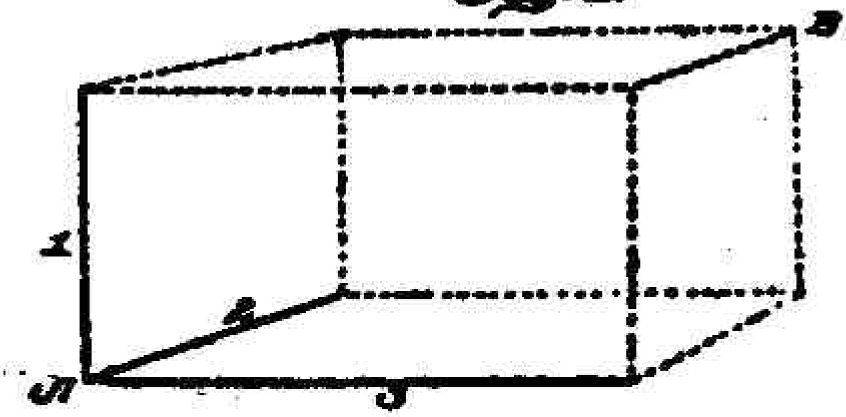
The Fourth Dimension

By C. H. Hinton

We know that every object has three dimensions, which are usually termed height, breadth, thickness. To take the simplest instances of these three dimensions, and the axes by which they are represented, consider the case of a room (Fig. 1). The lines 1, 2, 3 can be referred to as defining the directions of "up and down," "away or near," "right and left." We can proceed from one corner of the room to another by a straight line, such as AB. But we can equally well pass from A to B by going parallel to each of the axes in turn for a suitable distance. We can pass from any one point in the room to any other point by a combination of movements in these three directions. Since the room indefinitely expanded would occupy the whole of space as we think of it, we ordinarily assert that there is no point in the whole of space which we cannot reach by a combination of movements in three directions.

By means of movements in two directions we can only reach the points in a limited region of space: for instance, by means of movements parallel to axes 2 and 3 we can only reach points on the plane of the floor. But with all three axes and liberty of motion in three directions we can reach any point of space, as we conceive it.

Fig. 1.



AXES OF HEIGHT, LENGTH, AND BREADTH

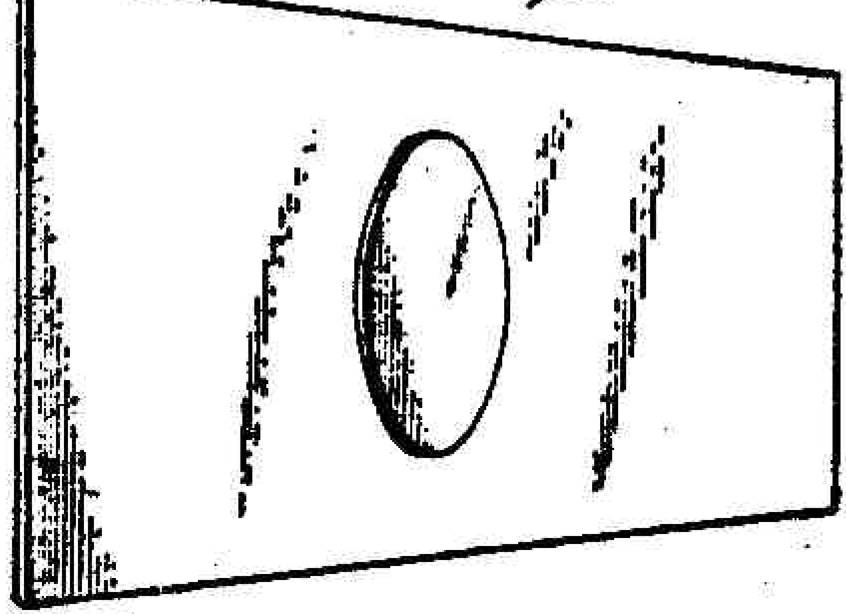
Some thinkers nevertheless have decided that the three dimensions, height, length, breadth, do not exhaust the possibilities of space. They say that just as motion involving the axes 2, 3 will not enable us to reach all points in the space of a room or of the room extended, so motions in the directions of all three axes will not enable us to reach all the points of space as it really is.

Hence the birth and growth of the idea called, for want of a better name, the Fourth Dimension.

Plato, at the beginning of the seventh book of the Republic, describes a set of prisoners who are held in chains before the mouth of a great cavern, bound so that they cannot turn their faces in any other direction than looking straight into the cavern.

On the wall in which the cavern ends they see their shadows projected by the sun. Their only experience of objects is derived by watching these shadows. If passers-by traverse the roadway behind them, all they see is the shadows of these passers-by on the wall. If an object strikes them, what they see is the shadow of that object striking the shadows of themselves.

Fig. 2.



A PLANE WORLD: THICKNESS EXAGGERATED

Plato draws the conclusion that they would identify themselves with their shadows. Since events occurring amongst these shadow forms are the invariable accompaniments of all their sensations, they would think that they themselves were those shadows, and lived and moved in a shadow world.

Now the shadows can only move on the surface of the wall; they cannot approach and recede from it. Hence the prisoners think of themselves as having a two-dimensional existence only. And, says Plato, as these prisoners think of themselves as less than they really are, so we in our turn think of ourselves as less than we really are. His philosophy was an effort to find that Greater which we really are.

Plato turns from the outward image to its inner significance, interrogating his self-consciousness. But in accordance with the modern habit of attending to the record of observation of the outward, let us trace out the objective experience of such prisoners.

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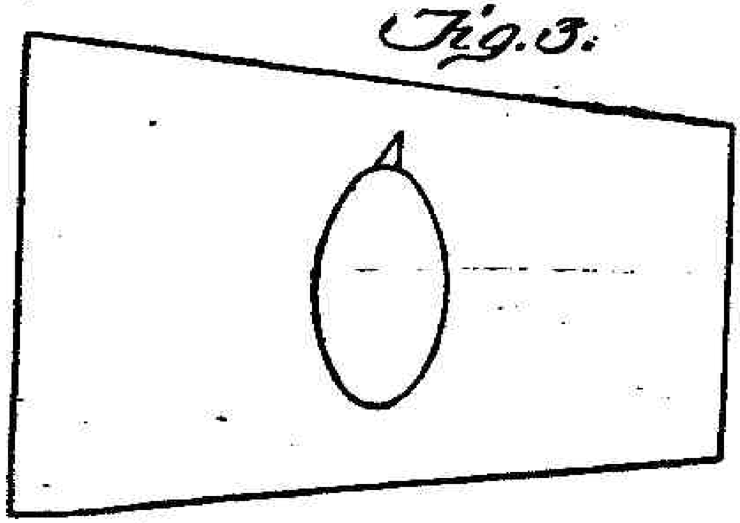
A being identifying himself with a shadow would think that the surface of the wall was all the space there was. The conception of a third dimension of space would be as foreign to him as that of a fourth dimension is to us.

By means of this retrogressive step of imagining a kind of existence in which experience is confined to less than our number of dimensions we are irresistibly led to propound the question: Is there a kind of existence dimensionally more ample than that of our experience?

Fig. 3.



A PLANE BEING ON THE RIM OF HIS WORLD

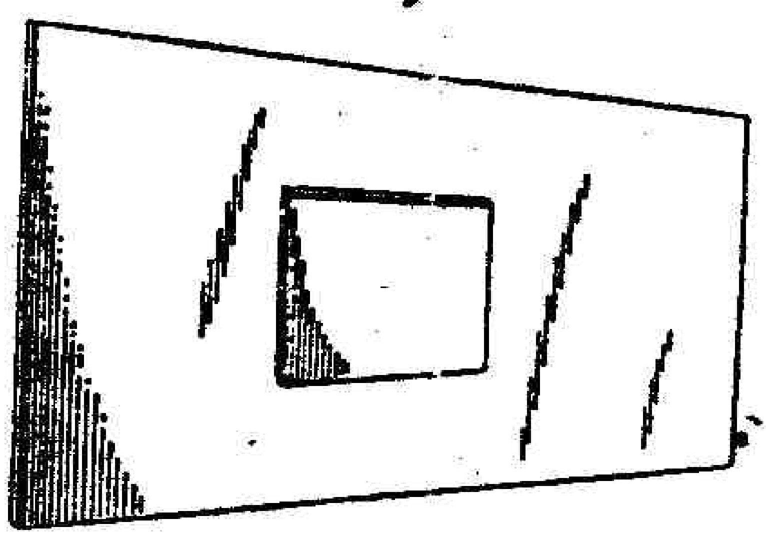
In his illustration, Plato avoids an error in which most modern representations of a plane world are involved.

He makes his beings real, not mere abstractions, such as geometrical beings in a plane. He places before us an imaginary scene in which real beings would have a conscious experience of a condition more limited than that of their actual existence. I will represent the same idea in a different manner, choosing my illustration so that it will give us the means of answering all questions that occur in the study of four-dimensional space, and will also lead us to an appreciation of the reasons for inferring its existence.

For this purpose imagine a globe to be cut in half, and of the half a thin slice to be taken. Imagine this thin circular disk to be placed against a great steel sheet over which it can slip perfectly freely; and suppose, moreover, that in virtue of some adhesion or attractive force the disk was held in contact with the sheet so as never to leave it (Fig. 2). This disk can be considered to be a "plane world."

Since it is material it should exercise attraction, and we must imagine the beings who inhabit it as standing on the edge. Let such a being be represented by the small triangle in Fig. 3. The force of gravity due to the disk would give him the direction of " up and down" (one dimension). Movement along the rim of the disk would give him the direction of "away and near" (a second dimension).

Fig. 4.



A SQUARE PIECE OF THE PLANE BEING'S MATTER; THICKNESS EXAGGERATED —

Now in order to have a being with a two-dimensional experience only, we must suppose that no movement other than in these two dimensions comes within the cognizance of his senses. Let us therefore suppose that the matter of which the disk and his own body are composed has only a very slight extension away from the disk. Let this extension be so slight that it escapes his closest observation.

Thus this "plane being," if he looked at any object composed of his matter, say a square figure, as in Fig. 4, would, as far as his consciousness was evoked, only see the length of the side opposite him. Since it is a real object, in order to come with in the sphere of his senses it must have extension away from the surface of the sheet. But this extension he would not recognize, and although the figure is real he would speak of it as though it were a geometrical square. He would believe that if this square were indefinitely extended it would fill up the whole of space. We see that it would only cover an infinite plane surface.

A being such as here described would lack a sense which we have, namely, a sense of "Right and Left." We see that a square figure made out of his thin matter has two faces, one opposed to the sheet, one turned away from it. But he

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would not know of these two faces. He would think of the square figure as totally accounted for by its height and length. We recognize that a two-dimensional existence is an abstraction. To be real an object must have all three dimensions. Thus his conception of his matter is false. His words refer to real things, but the thoughts which he connects with those words are thoughts about abstractions. The square figure, as he thinks it is, is not a real object at all. I speak of the plane being as if he were real, but a little consideration will show that no structurally organized being of this kind could exist.

For instance, since the thickness of his matter is less than any discernible extension, any canal or channel of visible size would divide his body into two disconnected portions. No circulatory system or alimentary canal would therefore be possible in his case. In fact, if we admit the existence of an ascending scale of dimensions, a being which has extension of comparable magnitude in each of three dimensions, as we have, is the first of beings in this ascending scale that can possibly exist.

The mechanical possibilities of a plane being would be of a very limited range. Two such individuals meeting, one would be obliged to climb over the other in order to pass him.

The very difficulty of apprehending the extreme limitation of such a being shows how great the step would be which he must take in order to apprehend our possibilities.

In like manner, even to state the possibilities of a four-dimensional existence would appear naturally to us in our turn a task of great difficulty.

But the beauty of the illustration I have given is that it enables us to settle every point that comes up in the imaginary construction of a four-dimensional world, by attending to the corresponding step that the plane being would have to take in forming his imaginary construction of a three-dimensional world.

I would therefore ask the reader to try to imagine himself as a very flat being moving along the rim of the disk, looking only before him and up and down, only able to reach out in front and upwards, not laterally, and only to be able to approach and recede from the objects on his earth in one plane of motion.

If one of us were to try to explain to a plane being the nature of our existence, we should have to introduce some new words into his vocabulary, and these new words would not have reference to anything that the plane being could see or point to.

If there is a fourth dimension of space, we must necessarily introduce words which have no reference to anything within the range of our conscious experience.

There must be some reason why we do not move in this fourth dimension, as there is a reason why the plane being cannot move "right and left."

In order to introduce the ideas that we have to form in the most simple way, let us suppose that there is a substance (analogous to the sheet against which the plane being slips) along which we slip freely in every movement we make, and against which every portion of our matter slips in every movement it makes.

Consider for the sake of simplicity that a square figure of the plane being's matter is made up of a number of particles forming a single layer against the plane.

The plane being would have to admit that every particle in such a square figure of his matter was as close to the sheet as every other particle, and although the interior of the square would only be approached by him by breaking through the bounding-lines, still from each point of the interior a line could go "right and left" in an unknown direction that he could not possibly conceive.

Similarly, on an analogous supposition, if we look at a cube of matter which is perfectly bounded by its faces in every way in which we can approach it, we must admit that there is a direction going off from every particle in the cube, and that it is possible to draw a line from each particle in each of the two opposite ways in this unknown direction.

The cube would be perfectly free to move in either of these opposite ways in the unknown dimension unless by reason of some constraint.

Let us call the hypothetical substance which is next to every particle of our

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matter in this unknown direction the "paron" (from "para," alongside, and "on," being). The paron corresponds to the sheet of which we should have to tell the plane being.

As we have to tell the plane being of an unknown extension of his matter to the right away from his sheet, so we have to admit that our matter has an extension away from the paron in the unknown direction. Let us call the direction in which our matter extends away from the paron the "apo" direction (apo meaning away), and call the opposite direction from our matter towards the paron the "eiso" direction (eiso meaning within).Then "apo" and "eiso" correspond to the words " right" and "left" which we should have to teach the plane being to use. As in his case, so in ours, there is nothing in our conscious experience which corresponds to these words. They have reference to an unknown direction, and by attending to the possibilities which such a new direction gives we can gain the means of putting the question rationally as to whether it exists or not.

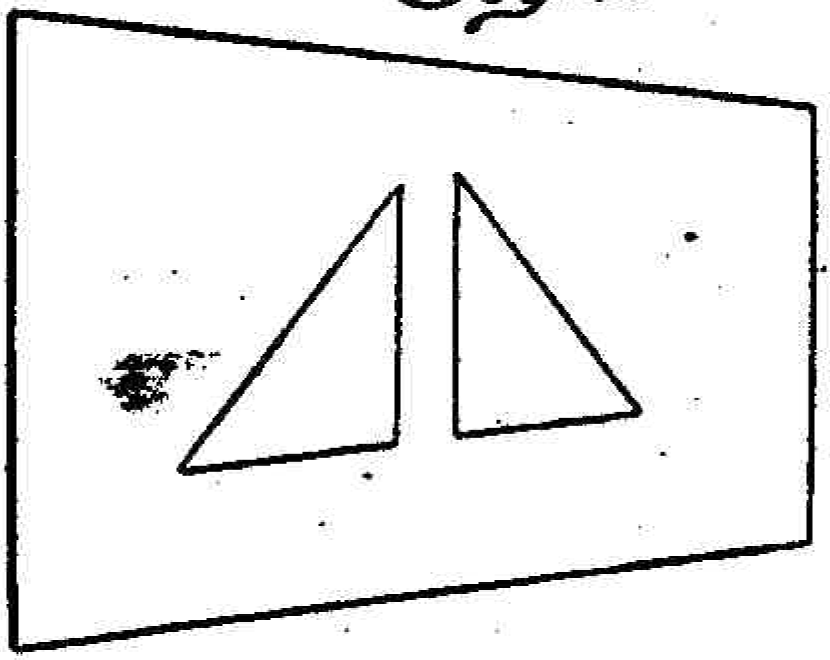
I will now briefly describe three cases in which an attempt has been made to find evidence for the reality of a fourth dimension. Cases 1 and 2 are such as would obviously suggest themselves to any inquirer.

Case 3 I shall also merely touch upon, as its general argument has been published (see *Bulletin* of the Philosophical Society of Washington, April, 1902), while the mathematical method used was exemplified in a paper printed in the Proceedings of the Royal Irish Academy, November, 1902.

Case 1 depends on the properties of configuration. In a plane three points can be found equally distant from one another, such as the vertices of an equilateral triangle. In our space four such points can be found such as the vertices of a tetrahedron. In four-dimensional space five points equally distant from one another can be found.

Now to account for the properties of organic compounds it has been necessary to assume that the carbon atoms in, the molecules of certain substances are related as the four vertices of a tetrahedron. If it became necessary to assume the existence of five atoms at equal distances from one another in a molecule, there would be evidence of a fourth dimension.

Fig. 5.



TRIANGLES WHICH, IF KEPT AGAINST THE SHEET, CANNOT BE MOVED SO THAT ONE WILL OCCUPY THE SPACE OF THE OTHER

Case 2 depends on the properties of rotation in four-dimensional space. We are familiarly acquainted with right and left handed shapes. The right hand itself and the left-hand image it meets in a mirror are examples of these configurations - they are alike, one another, on opposite sides of a plane. One cannot be turned into another in our space. Now in a plane, rotation takes place round a point; in our space, round an axis, hence we should conclude by analogy that in four-dimensional space rotation took place round a plane. This conclusion is found to be justified if one looks into the matter. In fact, in a plane two triangles such as shown in Fig. 5, are incapable of being turned into one another by any motion in the plane. One cannot be made to occupy the space of the other however it is turned about in the plane. Such figures correspond to our right and left handed shapes, and the rotation round a line by which they would be turned into one another is just as inconceivable to a plane being as rotation round a plane is to us. Our right and left handed shapes are, on the hypothesis of a fourth dimension, shapes turned half-way round.

Now there are two substances, two varieties of tartaric acid, which are alike in all physical and chemical properties, save in their behavior with regard to polarized light. One turns the plane of polarization in one direction, the other in the opposite direction. This is due to the molecules of the one being of exactly

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of the same configuration as the molecules of the other, save that those of one are right-handed and the other left-handed.

These two varieties change one into the other apparently without any chemical resolution and reconstitution. If such were certainly the case, it would be a proof of the fourth dimension, because only in four-dimensional space can a right-handed shape become a left-handed shape by simple movement.

Fig 6

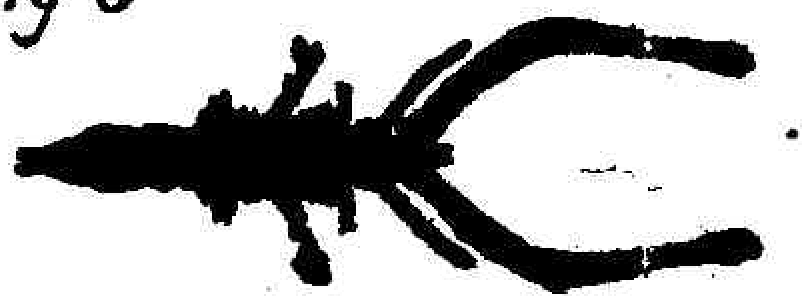

FIGURE SYMMETRICAL ABOUT A LINE

Case 3. - In the proceedings of the Washington Philosophical Society, November, 1902, the writer argues as follows: It is not at all certain that the mechanics which is found capable of explaining the processes of nature that occur on a large scale is capable of explaining what occurs in the minute. Right and left handed shapes never occur in any phenomenon on a large scale, such as rocks, clouds, configurations of continents, but do frequently occur as results of processes which take place in the minute, such as the vital processes, crystallization, etc. Now to produce figures symmetrical about a line the simplest way is to use a three-dimensional process. This is exemplified in Fig. 6, which was produced by folding over a piece of blotted paper, and in this folding over the third dimension was used.

Thus symmetry in two dimensions is produced by a three-dimensional process.

Similarly, it is not unnatural to expect shapes symmetrical about a plane as the result of a four-dimensional process. Thus it is worth while to form a complete system of four-dimensional mechanics, and it is only when such a system has been elaborated that we shall be in a position to determine whether the obscurities found in the domain of molecular physics are to be attributed to the complexity of the three-dimensional conditions or to the presence of four-dimensional motions. For example, no satisfactory explanation has been given on mechanical principles of an electric current.

To take the most familiar type, an electric current involves the existence of a wire or other conductor. The action is not conveyed through the wire, but, as Professor Poynting has shown, through the medium in which the wire is situated. It is therefore not incorrect to say that a continuous electric current is a disturbance in a medium which demands for its existence a continuous boundary (the conductor) in that medium.

Now in three-dimensional mechanics a certain type of disturbance is known which demands for its continuous existence in a medium that its opposite ends shall impinge on a boundary of the medium. Of such a kind is a vortex, which may be thought of as an eddy. A smoke ring is an instance of a vortex with its ends joined together. In a perfect fluid a smoke ring could exist with ends free from one another if these ends impinged on a boundary of the fluid.

Here we have the phenomenon of a disturbance involving as the condition of its continuous existence that its ends impinge on two opposite boundaries of the medium in which it takes place. This differs from the electric current because only two ends necessarily impinge on a boundary of the medium, while in the electric current a whole contour must impinge on a boundary.

Now examining into the nature of a four-dimensional vortex it is found that in such a disturbance of a medium the condition of its continuous existence is that it impinges on a boundary by a whole contour. Where a three-dimensional vortex requires two opposing boundaries a four-dimensional vortex requires a complete circuit of boundary. Thus a four-dimensional vortex has a striking analogy with an electric current.

It is in the examination of questions such as these that the physical inquiry as to the existence of the fourth dimension consists in asking, namely, whether the types of action which occur can be explained on the principles of three-dimensional mechanics, or whether they demand for their explanation the assumption of a four-dimensional motion.

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