Twist 'til we tear the house down!

By James E. Beichler

PART III

IV. The New Century and a New Physics

The turn of the century brought no magical changes in the world of non-Euclidean geometries. If anything, it offered a unique opportunity for everyone to reminisce on the changes that had taken place within geometry and mathematics in the previous century. At this time, scholars documented the fact that mathematicians were leaning toward giving credence to the possibility of a physical interpretation of the non-Euclidean geometries. Edward Kasner wrote about the changes from "attempts to discover universal methods" and develop an "ultimate geometric analysis," such as the quaternion analysis, to a more modest search for different theories of geometry.[141] He further confessed that it was the duty of mathematicians to study the "geometric foundations of the various branches of mechanics and physics."[142] It is obvious that he was not speaking of the strictly Euclidean basis of science. Corrado Segre, an Italian mathematician well known for his work in non- Euclidean geometries, expressed similar sentiments.[143]

Mathematicians were seriously considering their scholarly right to use the non-Euclidean geometries to represent physical phenomena. The trend was toward the acceptance of a physical connection between the mathematical and physical studies of non- Euclidean space, but the scope and method of application were ill defined. Federigo Enriques went still further in his 1906 book on the *Problems of Science*. He explained the connection between physical space and geometry, both the Euclidean and non-Euclidean varieties, and then stated that geometry was the basis of mechanics.[144] He did not distinguish between which geometry formed the basis of physical reality, but left the clear impression that he was fully willing to accept that physical space was non-Euclidean if that hypothesis was found necessary.

> In the present state of our knowledge, physical space must be positively regarded as Euclidean. But this does not justify the assertion that matter could not be otherwise. And it is unjust to accuse the non-Euclidean geometers of having raised a doubt, which is only removed for the present, and perhaps postponed to a distant future.[145]

Enriques would have had no reason to believe that the future date of which he speculated was only one decade away. Although he was a mathematician, his book was quite explicit in the explanation of physical theories.

A Treatise on Electrical Theory and the Problem of the Universe was still more explicit on current physical theories. Although G.W. de Tunzelmann of England published it in 1910, it provides a unique window on the physical attitudes of British science immediately prior to the development of special relativity. Relativity in a broader sense than expressed by Einstein was discussed, but only with regard to the theories of Henri Poincaré and H.A. Lorentz. De Tunzelmann made no references to either Einstein or Minkowski in spite of the fact that he made copious use of recent publications and scrupulously documented his references.

De Tunzelmann also offered a unique suggestion that time could be represented as a fourth dimension and explained the fundamental aspects of such a physical model.[146] After a discussion on absolute space, he also professed that an elliptic geometry fulfilled the necessary conditions for experiential space.[147] Although common three-dimensional space was completely relative, absolute space could be determined relative to an ether associated with a fourth dimension.

> When we think of space as filled with something, such as the ether, it seems to be much easier to think of position or direction relatively to it, even if we think of the ether only as a perfectly uniform continuous medium; and it becomes easier still when we think of space as full of ether whirls or spins which have to be traversed in moving from point to point.[148]

This model of an elliptical space was quite crude, but the source of de Tunzelmann's thought is not difficult to locate. The terms "whirls and spins" are reminiscent of Clifford's *Elements*. This fact should come as no surprise, since de Tunzelmann had been a student of Clifford four decades earlier.

It may not be fair to draw the conclusion that de Tunzelmann's thoughts on this matter reflected a general sentiment among scientists. But then, it is not necessary that scientists and scholars completely rejected Clifford's ideas at this date to demonstrate the influence of Clifford's work on the acceptance of general relativity. Just the fact that many were already familiar with Clifford's concepts of space immediately prior to 1915, disregarding their denial or acceptance, is adequate to indicate the influence of Clifford's work. It would be an unexpected bonus to prove that scientists fully believed Clifford's model of elliptic space represented reality, but that cannot be accomplished. Einstein presented a theory that Clifford reputedly was unable to develop and Einstein derived physical consequences of that theory which could be experimentally verified. Their methods were clearly different as were their immediate goals. Clifford was trying to explain electromagnetic phenomena with gravitation a secondary consideration while Einstein explained gravitation.

The fact that the academic community in its larger sense was already familiar with the notion that matter might be expressed as space curvature introduced a palatability factor that was missing when Clifford introduced his "Space-Theory of Matter" in 1870. Yet the historical consequences go deeper than just the question whether curved space was more palatable in 1915 due to Clifford's "Space-Theory." Clifford's actual theory was only a small, albeit extremely important part of a larger trend in accepting the possibility of a physical non-Euclidean space. In many cases, Clifford's direct influence cannot be discerned and that is justly so. However, at the very least an indirect influence can be assumed since Clifford was the founding father of the English concept of a physically curved space.

In 1908, Hermann Minkowski presented his space-time model of Einstein's theory of special relativity. Until that time, Einstein's theory of relativity was just one among many in which the Lorentz-Fitzgerald formulas could be justified. Minkowski presented a model by which the world was non-Euclidean, not just four-dimensional. In a community where the possibility of a non-Euclidean space was already being considered, where many scholars would not admit that our experiential space was Euclidean simply because astronomical observations could not prove otherwise, Minkowski's model of space-time was not just four-dimensional, but implied a non-Euclidean four-dimensional structure for space-time.

Halsted wrote of the space-time model "the theory of relativity has made non-Euclidean geometry a powerful machine for advance in physics."[149] He specifically singled out the work of a Croatian mathematician, Vladimir Varičak, who was able to derive the equations of special relativity directly from his studies of Lobachewskian geometry.[150]

Henry P. Manning of Columbia University also confirmed the non-Euclidean interpretation of space-time. He characterized space-time as a "system [which] may be regarded as a non-Euclidean geometry in which the conical hypersurface plays the part of absolute angles, while distances along lines of the two classes are independent and cannot be compared."[151] Like these other men, Manning had a long association with the non-Euclidean geometries before the development of relativity theory. Manning's association with the purely mathematical studies of geometry did not overshadow his willingness to look at the physical interpretations of geometry.

In 1910, an anonymous donor gave *Scientific American* magazine five hundred dollars to hold an essay contest on the fourth dimension. The competition proved so popular that two- hundred and forty five essays were submitted from nearly every civilized country in the world.[152] The contest was judged by Manning and S.A. Mitchell of Columbia University. Manning published a group of the better essays in 1914 under the collective title *The Fourth Dimension Simply Explained*. He wanted to save these essays for posterity. Within the published essays, there was absolutely no mention of Einstein's relativity theory or Minkowski's space-time model, but there was ample evidence of the seriousness with which the non-Euclidean geometries and their physical counterparts were taken by the educated populace during the period of time immediately prior to Einstein's discovery of general relativity.

In the book in which he first mentioned relativity, a purely mathematical study of *Non-Euclidean Geometry*, Manning also mentioned the work of Gilbert N. Lewis and Edwin

Bidwell Wilson. In 1912, these two men collaborated on a non-Euclidean theory of relativity based upon the Minkowski model of space-time. They felt that "any line in our four-dimensional manifold which represents motion with velocity of light must bear the same relation to every set of axes" was "sufficient to determine the properties of" our non-Euclidean space.[153] Both men had some previous experience with non-Euclidean geometries, but Wilson's experience was quite extensive. As early as 1904, he had criticized the overly philosophic trends that were exhibited by many geometers. He thought that mathematicians had been displaying "a mania for logic" which was wholly unjustified and that there was nothing of reality behind this logic.[154] Something more was needed in geometry beyond the logic of axioms, something intuitive, perhaps a "postulate of reality."[155] From these observations, it is obvious that Wilson did not accept a complete distinction between abstract geometry and the real world.

Harry Bateman also developed an essentially non-Euclidean theory of space and time. In this case, the theory preceded even Minkowski's space-time by a short time.[156] Bateman worked on expressing electromagnetic waves by a geometry of spheres in his four-dimensional space with time as a fourth dimension, a situation analogous in many ways to a non-Euclidean geometry. Bateman, who had some previous experience studying pure rotations in a four-dimensional space, developed the mathematics of general covariance by 1910,[157] a feat not accomplished by Einstein using Christoffel tensors until several years later. Bateman did not claim that his geometry was non-Euclidean, but implied this description.[158]

Not only were non-Euclidean versions of Minkowski's space-time model being developed before general relativity, but Hans Kleinpeter remarked on the similarity between Clifford's concepts of space and time and Minkowski's space-time in his 1913 German translation of Clifford's *Common Sense*.[159] Kleinpeter's note to this effect appeared on the page preceding Pearson's original editor's note relating space curvature to physical phenomena and the twist to magnetic induction. It is unlikely that Kleinpeter, a German, was the only person with knowledge of Clifford's most popular published work to draw this analogy.

Perhaps the earliest public mentions of Clifford's work in conjunction with general relativity came at the hands of Ludwik Silberstein in 1918. Silberstein did not fully accept general relativity as written, but investigated its tenets and consequences. In particular, he considered the theory without the principle of equivalence. In the course of this study, he noted that Clifford had already equated curvature with matter.[160] The fact that he mentioned this is not so important as the context. His attitude was that equating curvature to matter should not be regarded as a new accomplishment. Clearly, he would not have given Einstein credit for this particular advance in science, but would have awarded Clifford the honor.

Silberstein compared general relativity to the "Space-Theory" and *Common Sense*, but other writers made early comparisons with Clifford's other publications. Henry L. Brose recommended that readers of his translation of Erwin Freundlich's *Foundations of Einstein's Theory of Gravitation* refer to Clifford's article on "Loci" and H.J.S. Smith's introduction to Clifford's *Mathematical Papers*.[161] Sir Oliver Lodge, by no means a supporter of general relativity, attempted to explain away the positive results of the light bending measurements by arguing that either the ether near the sun changed the refractive index of space or the ether composing the light beam reacted to the gravitation of the sun. Only if these hypotheses could be decisively refuted, could Einstein's theory be considered. He then referred to Clifford's "Philosophy of the Pure Sciences."[162] Even then, general relativity was only a mathematical gimmick to give the correct experimental results, and was only palatable since Clifford had already shown the comparison of ether and curvature, or so Lodge implied by his reference to Clifford's work. But only those scientists, who were familiar with Clifford's work, as were the British scientists of that era, would have recognized the implication. So the implication is lost to anyone reading Lodge's paper today.

Neither de Tunzelmann[163] nor Bateman referred to Clifford in their limited adoptions of relativity, but neither left any doubt that their own preconceived notions of space curvature limited their acceptance of general relativity. In de Tunzelmann's case this proceeded directly from Clifford. On the other hand, Bateman's references to general relativity were especially significant. Bateman thought that he had discovered the same theory several years before when he discovered the "general principle of relativity," the general covariance under all transformations.[164] There might be some small amount of legitimacy to this claim. Some scientists who first adopted relativity considered the "general principle of relativity" as the more important aspect of Einstein's theory rather than the expression of space curvature as matter. This aspect of the development of general relativity would explain why Silberstein gave Clifford rather than Einstein credit for equating space curvature to matter. Willem de Sitter had noted this very fact in his 1916 article on "Space, Time, and Gravitation" in *The Observatory*.[165] If the "general principle of relativity" were considered the more significant part of Einstein's theory at this early date, then Clifford's priority for equating matter to curvature would be preserved and the early references to Clifford's other works explained.

But it was the work of Sir Arthur S. Eddington, who led the expedition to confirm Einstein's light bending prediction, which so clearly demonstrates the greatest influence of Clifford. Eddington became intrigued with general relativity after reading de Sitter's 1916 accounts of the astronomical consequences of the theory.[166] In his earlier publications on the theory, Eddington indicated that he did not fully believe in the literal truth of space curvature.[167] His early interpretations of the theory were decidedly Victorian with talk of strains in the ether, but Eddington's ability to handle the different non-Euclidean concepts as well as his perspective on the theory developed very rapidly and continuously. He admitted that he originally knew little of the non-Euclidean geometries,[168] so it can be concluded that he made a study of the non-Euclidean geometries to fill in the gaps in his own knowledge of the subject.

It is quite likely that his basic concepts on the non- Euclidean geometries came from Clifford. If he didn't already know of Clifford, he must have become very interested in Clifford's work because he was able to show a great familiarity with Clifford's work in just a few years. In his 1921 popular exposition of the theory, *Space, Time, and*

Gravitation, Eddington introduced one chapter by a quote from *Common Sense*[169] while he began the chapter on "Kinds of Space" with a quotation from Clifford's "Postulates."[170] The quote from *Common Sense* was the same paragraph that ended Clifford's chapter on "Position," and the very words to which Pearson added the note that twists may well represent magnetic induction.

However, Eddington also quoted a passage from the "Unseen Universe" in which Clifford expressed his desire that physical reality would one day be expressed as the geometry of position. "Out of these two relations [nextness or contiguity of space and succession in time] the future theorist has to build up the world as best he may." What might help the scientist in this endeavor, suggested Clifford, was the description of distance as an expression of position as in the mathematics of 'analysis situs' and the fact that space curvature could be used to describe matter in motion.[171] It was implicit in Clifford's original context of this statement that the ether could be replaced by space curvature for a total theory of the physical world of matter. Eddington's first work on general relativity clearly displays his Victorian heritage and education. But as his ideas about general relativity evolved beyond Eddington's Victorian bias, Clifford's words and influence seemed to exert an ever-stronger presence in Eddington's own work.

Two and a half decades later, E.T. Whittaker wrote a history of scientific conceptions of the external world, *From Euclid to Eddington*. The book ended with a statement that Eddington was attempting to reduce all of physics to "one kind of ultimate particle, of which [the known elementary particles] are, so to speak, disguised manifestations."[172] A comparison of this with Clifford's goal, as expressed in the closing remarks of the *Elements*, indicates that Clifford and Eddington's goals were essentially the same, the physical expression of the universe based upon the various manifestations of a single particle. But their methods of achieving that goal were quite different. Eddington did not use Clifford's twists, but did adopt Clifford's basic philosophy as well as borrow some of Clifford's mathematics. Regarding the similarity between their philosophies, Smokler even suggests that Eddington's book *The Nature of the Physical World* be referred to for an explanation of Clifford's philosophy.[173]

The theory to which Whittaker referred was Eddington's "fundamental theory." Eddington had already presented various papers and articles on the theory and these were collected, edited and published by Whittaker in 1946, after Eddington's death. The fundamental theory was meant to be the pinnacle of Eddington's considerable work and long association with the theories of relativity, the quantum theory and cosmology. The theory was based upon the mathematics of E-numbers, which represented the elements of an E-frame that Eddington associated with our physical space-time. This E-frame, in conjunction with an F-frame to which it was related, then allowed a new interpretation of the Christoffel tensors from which Einstein had constructed his own mathematical model of space-time curvature.

The E-numbers were quaternions and shared many characteristics with both Clifford's biquaternions and Ball's screws. But Eddington's application of quaternions was different because the essential problem of finding a mathematical model was different for Eddington than it had been for Clifford. It had become necessary for Eddington to account for all of the physical concepts and phenomena that had been discovered since Clifford's death: quantum theory, the Bohr atom, radioactivity, the atomic nucleus, electrons, protons, neutrons, the theories of relativity and others. So Eddington's theory was different from Clifford's even though they were philosophically similar and could not be considered a simple continuation of Clifford's work.

In 1944, Eddington published a paper entitled "The Evaluation of the Cosmical Number." He had intended that the ideas presented in this paper be included as the epistemological basis of his theory in its final version. So Whittaker added the paper as an appendix to the posthumous publication of the *Fundamental Theory*. In his epistemological explanation, Eddington stated that the central problem that he had addressed was "to discover a structure of measures and measurables which is such that this promise [of distinguishing between measures and measurables] can be fulfilled."[174]

Measures, which are strictly geometrical in nature, formed the basis of Eddington's model of space-time, while measurables could be interpreted as purely physical particles. It was necessary that both contribute to the structure of space-time even though they had to be distinguished one from the other at the same time. The problem for Eddington was that measures and measurables were both the same and different. The science of space-time was thus reduced to a question of distinction between the two.

> The data of physics are measures; but we can make nothing of a mere collection of measures without any note of the objects and circumstances to which they refer. The crux of the problem is to supply 'connectivity' to the measures; so that in the theoretical treatment there may be an equivalent for that part of the procedure of measurement which consists in noting the objects and circumstances to which the measures relate.[175]

From this statement alone, Eddington's philosophical debt to Clifford is clearly evident. Eddington's measurables were in a very broad sense the same as Clifford's twists. The problem faced by Clifford in discovering the mathematics of ‘connectivity’ between the individual contiguous points of space was the same as those described by Eddington. This problem lead Clifford to the development of that particular non-Euclidean geometry which he had hoped to use to describe the 'structure' of space in his own space-theory of matter, just as it lead Eddington to the development of his own fundamental theory. If Eddington had used any other word than 'connectivity,' which he himself had emphasized, the case for Eddington's debt to Clifford would have been harder to make, but not impossible to make. But the idea of 'connectivity' was so essential and unique to Clifford's mathematical development that this word alone proves Eddington's debt to Clifford. This single statement reflected Clifford's concepts as much as Eddington's.

Eddington made no reference to Clifford's earlier work nor would he have been compelled to cite Clifford as the source of his ideas since his own theory was far more comprehensive than Clifford's and thus quite different in application. Also, Clifford's concepts had long been accepted as part of the public domain of science so there was no need to cite Clifford directly. What can be determined from these examples with historical accuracy is that Eddington's views on science and the physical world, from the very beginning of his association with general relativity until his death, if not before 1916, were profoundly influenced by Clifford's earlier researches and conceptual developments. This influence could not have been unique in Eddington's experience alone, but would have been true for many others.

For his own part, Whittaker made no reference to Clifford in his book *From Euclid to Eddington* and only briefly mentioned Clifford in his *History of the Theories of Aether and Electricity*,[176] but this oversight is insignificant. By the time that these books were written, Clifford had already received adequate recognition by many scientists as the originator of the concept of matter as space curvature as well as inaccurate recognition as the "anticipator" of general relativity. Actually, what Clifford had anticipated went well beyond just the use of space curvature as matter as described in general relativity.

Nor did Thomas Greenwood directly mention Clifford in his 1922 essay "Geometry and Reality," even though Greenwood did relate other interesting facts regarding the general attitude toward space curvature. After explaining that astronomers had been searching for space curvature for some time by careful observation of stellar parallax, Greenwood continued to describe another aspect of non-Euclidean science that was common knowledge before relativity.

> But all these [parallax] observations proved negative: space presented itself as Euclidean. Nevertheless there was an idea amongst men of science, that more accurate observations and the development of mechanical consequences of non-Euclidean geometry with regard to astronomical problems, would certainly favour the legitimacy of non-Euclidean postulates as physical hypotheses.[177]

These simple historical facts, as explained by Greenwood, seem all but forgotten by modern historians and scholars who study the genesis of general relativity.

Clifford had translated Riemann's work into English. He was the first scientist in the English speaking world to describe the problem of parallax measurements with respect to space curvature before the public and the first to popularize the concept of non-Euclidean geometry in his presentation of "The Aims and Instruments of Science" and the "Philosophy of the Pure Sciences." His "Postulates" was considered a classic of science by the turn of the century, as was his *Common Sense*. Clifford was the founder of mathematical studies on the dynamics of non-Euclidean space and discovered a whole class of non-Euclidean geometries. These were no mean accomplishments and Greenwood did not need to mention Clifford's name within the context of the pre-relativistic search for a physical non-Euclidean space. When he referred to the mechanics of motion in a non-Euclidean space, he could have been speaking of no one but Clifford.

By the date of general relativity's initial development, Clifford's ideas had been disseminated throughout science and culture and in many cases were no longer associated with Clifford's name. Although the suggestion that space curvature could have physical consequences can be attributed to Riemann alone, only Clifford had gone so far as to link small variations of curvature with the concept of matter itself and begin the task of redefining the very concept of force in terms of such a space curvature. It is also quite evident that a continuous historical line can be found flowing from Clifford's initial ideas through the work of Hinton, Robert Ball, Pearson and others, to the more generally held belief by many scientists and the common educated populace that the real physical space of human perception could possibly be non-Euclidean. This was an accepted fact on the eve of the discovery of relativity theory, as Greenwood implied.

On the other hand, there are no causal links between the mathematical theory of relativity and Clifford's mathematical researches because the mathematization of space curvature did not follow the path originally explored by Clifford. Under Einstein's direction, the mathematical model of space-time curvature was to be based upon tensors rather than quaternions. So whenever historians and scholars have seen fit to trace the historical roots of general relativity into the past, they have generally followed the concrete examples of the mathematical lineage rather than the more spurious development of concepts and attitudes deriving primarily from Clifford's work. Such a "quick fix" of history does not tell the whole story. The development of tensor calculus was a purely mathematical exercise, devoid of physical content. So it would seem to anyone tracing the mathematical development of tensors that the space-time curvature represented by tensors was devoid of physical interpretation before Einstein's work was completed and thus accept the fact that Einstein was the first to give a new "physical meaning" to the purely mathematical model of curved space based on tensors. This seems to be true at least for the case of Einstein himself, for whom no evidence exists of a previous knowledge of either Clifford's "Space Theory of Matter" or Clifford's other physical interpretations of space curvature. But it must be remembered, and rightly so, that the new theory of general relativity was grafted on to an already considerable and growing recognition of the fact that space could well be and probably was non-Euclidean.

For many of those who were interested in the scientific problem of space curvature before general relativity, this attitude was supplemented by a previous knowledge of Clifford's physical concepts of mass and force. So we have such nonchalant statements as that made by Frank Kassel in his 1926 doctoral thesis that the "principle [which demonstrates that Euclidean geometry should be abandoned with general relativity] is an outcome of a thought emphasized by Clifford: that, namely, the metrical properties of space are wholly determined by the masses of bodies."[178] Kassel's statement came a decade after the inception of general relativity and he drew no historical connections between Clifford and Einstein, but neither did he hesitate to associate their ideas on a purely philosophical level. Even then, neither the physicist E.H. Kennard nor the philosopher Edgar H. Singer of the University of Pennsylvania must have objected to this association of ideas. If they had objected, Kassel's statement would not have accepted in the published text of his thesis. Many such statements can be found in the decade following 1916 which would lend further support to the conclusion that many of the scientists and scholars who first accepted Einstein's theories had a previous knowledge of Clifford's work.

There is no way to absolutely "prove" that these scientists and scholars accepted general relativity "because" they had a previous knowledge of Clifford's concepts of matter and space, and it may well be inaccurate to even voice such an opinion. But there is certainly a preponderance of evidence indicating that Clifford's work influenced the following generations in such a way that, in essence, he laid the foundations for the positive attitudes toward physical interpretations of curved space upon which general relativity later built its own following and found a comfortable home. Therefore, the acceptance of general relativity by the scientific community was enhanced and accelerated by the previous knowledge of Clifford's work. There was no longer a need for the numerous philosophical arguments against space-curvature that had plagued Clifford's original ideas, so such arguments did not develop after the advent of general relativity. This is especially true in England and America where Clifford's concepts remained popular throughout the years between his death and Einstein's success.

Clifford was a major player in opening a whole new field of scientific inquiry in which our basic notions of space, time and force and their relationships to electromagnetism and gravitation were challenged, even unto this day. Even the recent theory of "twistors" in which Roger Penrose attempts a grand unification of the natural forces is based upon Clifford's earlier work. By introducing the concept of a "twist" as an element of space curvature, Clifford began an intellectual movement to tear down the house that forms our preconceived prejudice toward a physics based solely upon Euclidean space and replace it with a more general concept of space curvature which could account for both gravitation and electromagnetism.

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PART II
ENDNOTES

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