
Twist 'til we tear the house down!

By James E. Beichler

PART II

III. The Followers

Clifford must have felt a great deal of gratification in 1877 when Frederick W. Frankland's essay on non-Euclidean space appeared in *Nature*. Before moving to New Zealand for reasons of health, Frankland had been a student of Clifford. The paper was an effort to study the characteristics of a special type of Riemannian or elliptic geometry, but only for the case of two dimensions. Frankland had originally presented the essay before the Wellington Philosophical Society in November of 1876. It was subsequently read before the London Mathematical Society before publication in *Nature* in April of 1877.⁶⁶ A similar geometry was investigated by the American astronomer, Simon Newcomb, with the results published in the German journal *Crelle's* in 1877.⁶⁷ While Frankland's presentation was more philosophical, tracing the logical development of a curved two-dimensional surface, Newcomb developed the purely mathematical characteristics of a similar three-dimensional curved surface. Klein had discovered this type of surface, which later came to be known as the single elliptic or polar form of Riemannian geometry.⁶⁸ Newcomb's discovery was independent of Klein's and Newcomb has been given credit as co-discoverer of this geometric system.⁶⁹

Given the date of Newcomb's publication, it is possible that Clifford's work influenced Newcomb's research. Newcomb had traveled to England before the publication and it is quite possible that he met and spoke with Clifford, the "Lion of the season"⁷⁰ on his visits to London. Otherwise, there are enough references to Clifford in Newcomb's later publications to conclude that it would be wrong to think that Newcomb had never been influenced by Clifford's thoughts. After the turn of the century he referred to Clifford as the only person who had ever truly understood gravitation,⁷¹ implying that he had a more intimate knowledge of Clifford's thoughts than could be gleaned from Clifford's publications.

When Frankland's paper was published it initiated some small controversies. In New Zealand the whole concept was attacked⁷² and in England Monro noted a few small points of difficulty in the pages of *Nature*.⁷³ After reading Newcomb's paper and checking Clifford's *Elements*, the difficulties experienced by Monro were resolved.⁷⁴ Monro raised an even greater question⁷⁵ over Newcomb's second paper on the non-Euclidean geometry, published in the *American Journal of Mathematics* in 1878.⁷⁶ In this paper, Newcomb proved that a hollow sphere could be turned inside-out without tearing or rupturing its surface by transiting a single elliptic space. Monro published, under

Cayley's sponsorship, a paper on this phenomenon in the *Proceedings of the London Mathematical Society*.⁷⁷ This paper constituted Monro's only mathematical publication on the non-Euclidean geometries. Newcomb published no other papers on the mathematical aspects of the non-Euclidean geometries, but returned on several occasions to popular expositions of them as well as commenting on them from time to time in other publications and presentations.

However, Frankland's paper inaugurated a more lengthy study of the possibility of explaining physical phenomena by space curvature. Frankland's researches were based, by admission, on Clifford's concept of the connection between contiguous points of space.⁷⁸ Frankland moved to America in 1892, settling in New York. Living in the United States where he found a more open and receptive audience offered Frankland a greater opportunity to discuss his theories with mathematicians and scholars. He presented his "Theory of Discrete Manifolds" at the summer meeting of the American Mathematical Society in 1897.⁷⁹ Newcomb presided over this meeting. Except for a short description of his presentation in the Society's *Bulletin*,⁸⁰ Frankland's theory was not published. Commentators complained that his theory suffered from obscurity from the failure to publish it. They could not evaluate the theory since they could not obtain copies of it. However, a collection of the dozen or so separate papers which constituted his theory were finally published in New Zealand in 1906.⁸¹ In spite of this publication, the theory still remained obscure and did not greatly influence the development of the non-Euclidean geometry.

Perhaps the greatest influence on the development of the non-Euclidean geometries in America was the arrival of Sylvester in 1877 as the professor of mathematics at Johns Hopkins University. He taught alongside Newcomb and Charles S. Peirce. His first student was George Bruce Halsted who became world renowned for both his contributions to the history of non-Euclidean geometry and his mathematical publications on geometry. Halsted privately believed that physical space was hyperbolic or Lobachewskian,⁸² but publicly he only admitted that it was impossible to distinguish which type of geometry was the true geometry of space.⁸³ Peirce actually developed a theory of hyperbolic space in the early 1890's. He thought that he had detected a discrepancy in parallax measurements between stars which could only be accounted for by assuming a Lobachewskian type of space.⁸⁴ Unfortunately, the lack of support for his ideas forced him to abandon the effort and his theory was discarded, never having been published nor committed to paper.⁸⁵

Sylvester also taught W.I. Stringham who continued Clifford's work on "Loci" and conducted a mathematical investigation of rotations in spaces of four dimensions.⁸⁶ It is no coincidence that the work of these men closely reflected the ideas of Clifford. Clifford's own essay on "Grassmann's Extensive Algebra" was published in the first volume of the *American Journal of Mathematics*, as was Newcomb's paper on the transformations of surfaces in spaces of four dimensions. The journal was founded by Sylvester and carried the stamp of his influence just as his influence generated the early American interest in the non-Euclidean geometries. It would be erroneous to think that Sylvester did not inform his American colleagues and students of Clifford's theory as

well as his own concept of geometrical reality while he was in America. Each of these American scholars had been influenced by Sylvester's tenure at Johns Hopkins, before Sylvester returned to a professorship at Oxford in 1885.

Of far greater importance were Karl Pearson's extensions of Clifford's work. In 1885, Pearson published the *Common Sense of the Exact Sciences*, which had been left partially completed by Clifford at his death.⁸⁷ When Clifford died the English academic community deeply felt the pain and loss of his passing. Even his detractors found kind words for him and expressed the great loss for England and society as a whole by his death. Ingleby wrote Monro that he took Clifford's death "to heart" and wished that he "had the brain of Clifford." He thought that the "death of Clifford might well throw all our churches into deepest mourning."⁸⁸ These private comments concerning Clifford's death are all the more important when it is considered that Ingleby had been Clifford's most vocal detractor in the earlier part of the decade.

The loss felt by scholars in England was exacerbated by the fact that Clifford failed to write down many of his lectures. Friends and scholars alike feared that his ideas might be lost to posterity. Hence, there developed a movement to publish anything and everything of Clifford's as quickly as possible after his death. This was a frantic effort to save the work that was considered too valuable to be lost to the world. Clifford's friends Frederick Pollock and Leslie Stephen collected and published Clifford's *Lectures and Essays*,⁸⁹ the papers which were to constitute the *Common Sense* went to Professor Rowe at Oxford while Robert Tucker collected Clifford's mathematical papers⁹⁰ and gathered together the fragments which were to become the second volume of the *Elements*.⁹¹ The extent of these endeavors was unprecedented and represented a tribute to the friendships that Clifford built during his life as well as the deep respect that his peers and colleagues had for his work.

Rowe died a few years after Clifford and the manuscripts for *Common Sense* then passed to Pearson. Pearson published the book early in 1885. *Common Sense* summarized Clifford's concept of space and time and offered a unique view of Clifford's method of mathematics as well as the best exposition of his concepts of curved space. Large parts of the *Common Sense*, including the section which described Clifford's concept of space curvature, were written by Pearson.⁹² But Pearson only meant to describe Clifford's concepts, not his own, and the work was accepted as an accurate portrayal of Clifford's ideas.

It is difficult to understand why all those authors who have studied Clifford's work have insisted upon the fact that Clifford had no followers while quoting passages written by Pearson in Clifford's *Common Sense*. A closer study of Pearson's scientific researches during the decade of the 1880's shows that Pearson was developing a theory of electromagnetism and atomism based directly upon Clifford's twists. He combined two strands of theoretical work, one on pulsating spheres of aether⁹³ and the other on twists,⁹⁴ to develop his final theory of "ether-squirts," published in 1891.⁹⁵ His theory was published in the *American Journal of Mathematics*, once again demonstrating the greater American tolerance for such ideas. The English mathematical community was already

beginning to slip into the doldrums of philosophical introspection, a movement that was largely a stepchild of the philosophical crises brought on during the earlier debates on the non-Euclidean geometries.

Pearson's ether-squirts were sources and sinks where ether flowed into and out of our space from a fourth dimension. The theory was a purely mechanical theory of the ether, rather than a mathematical theory of space curvature as Clifford had intended. In the publication, Pearson refused to speculate on the source of the ether in the fourth dimension, leaving that task for the transcendentalists.⁹⁶ He also made no public comments on the relation of his theory to space curvature, but privately he acknowledged that space curvature was the bottom line in the human perception of reality.

In a letter to his friend Robert J. Parker, written in 1885, Pearson commented that Kelvin's attempts to weigh the ether were conceptually erroneous, "as if empty space could weigh anything! I am going to weigh a twist!"⁹⁷ In this one private statement, passed between intimate friends, Pearson confirmed that Clifford's twist, which had been associated with the ether, was no more than an element of space curvature. Pearson had also commented on the Clifford's twist in a footnote in the *Common Sense*. In this case, he likened the twist to magnetic induction.⁹⁸ Although this suggestion was made in an editor's footnote, which would seem to suggest Pearson's personal opinion rather than Clifford's thought, the fact that Clifford considered this possibility was later confirmed by Charles T. Whitmell,⁹⁹ another student of Clifford's, and Frankland.¹⁰⁰

Pearson's development of a strictly mechanical theory of ether-twists over the period of a decade was accompanied by an evolution in his own philosophy and methodology of science. In December of 1885 he presented a talk on "Matter and Soul" before the Sunday Lecture Society. In this lecture, he described and evaluated the prevalent theories of matter: The Boscovichian atom, Kelvin's vortex theory of the atom and Clifford's space-theory of matter. Boscovich's atom represented no more than "non-matter in motion,"¹⁰¹ an absurdity, and was therefore rejected. Kelvin's vortex atom was "very like non-matter in motion" since stopping the motion would create a massless void.¹⁰² The possibility was not rejected outright, but severely questioned. On the other hand, Clifford's space-theory was non-mechanical. Matter was something in motion, but the something was geometric, the changing shape of space.¹⁰³ Boscovich's and Kelvin's theories were examples of how matter could be explained as a product of motion, while Clifford's theory sought to explain motion itself. Pearson concluded that matter could never be explained by a mechanical theory and Clifford's was the only non-mechanical theory available. This conclusion implied the ultimate superiority of Clifford's point of view.

Pearson further criticized the definition of mass. Since matter could not be explained by a mechanical theory, then mass could not be defined as the "quantity of matter," as it had been by Newton. Mass was merely the ratio of a force to the acceleration resulting from that force. Here, Pearson differed from Clifford who had defined mass as "stuff,"¹⁰⁴ but perhaps he was being too hard on Clifford. Clifford had stated his intention to redefine mass more precisely, but died before he could do so. Even

so, differences of opinion were beginning to show between Clifford and Pearson's concepts.

The evolution of Pearson's own ideas on the philosophical and methodological aspects of science ended with the publication of the *Grammar of Science* in 1892. The ideas that he expressed in this book were as similar to Ernst Mach's as they were to Clifford's. Space and time were reduced to "modes under which we perceive things apart," rather than realities in the world of phenomena.¹⁰⁵ Scientific concepts became limits extrapolated by our perceptions of the phenomenal world.¹⁰⁶ Mach's influence was evident in these attitudes rather than Clifford's. It seemed that Pearson had abandoned Clifford's notion that space curvature was the underlying reality, yet his new ideas still reflected the influence of Clifford's "mind-stuff" with a Machian turn (or twist).

Pearson had become disillusioned with the reception of his work on an ether theory of matter prior to his publication of the paper on ether-squirts. He had also been involved in a sometimes-frustrating debate with Kelvin over the ultimate existence of the ether.¹⁰⁷ In a letter to Kelvin, which was never finished or mailed, Pearson stated his final opinion on the matter of space curvature in a clear and concise manner, the like of which cannot be found in any of his published materials. He claimed that space curvature did not represent ultimate reality.¹⁰⁸ In this way he moved beyond his earlier opinions on the subject and decided that space curvature was only the final step toward a reality of which we could not have any physical knowledge. In terms of his statements on space in the *Grammar*, the representation of matter by space curvature was not reality, but the limit to which the human mind extrapolated its best and most precise perceptions of reality. In this sense, he had not so much withdrawn his earlier conviction to the reality of space curvature as he had decided that it was not possible for the human mind to have knowledge of ultimate reality.

When coupled with other events in his life, Pearson's disillusionment with the reception of his theory of matter gave him the opportunity to abandon any further attempts to realize Clifford's goal and relate his ether-squirts to Clifford's mathematical twists and space curvature. Pearson turned away from his past theoretical work and began working in the new field of the statistics of heredity which was far more rewarding. He never returned to his work on Clifford's theory, nor is he remembered for that work. Pearson's *Grammar* is still used to portray his philosophical ideals as well as the overall philosophical temper of science in the late 1890's while Pearson is quite well known for his work in statistics. In fact, Pearson is regarded as the father of modern statistics.

While Pearson worked toward his own theory of twists in the form of ether-squirts, Ball was developing a purely mathematical and analytical theory of screws in a non-Euclidean space. Each researcher attempted to continue Clifford's work in his own manner, but there was apparently no collusion between the two men. Their theories were characteristically different. Unlike Pearson, Ball made very few statements regarding the physical applications of his theory of screws, steered clear of the philosophical aspects of his work and never attempted to relate his mathematical researches to the ever-popular ether theories. Each of these men could look to different aspects of Clifford's *Elements*

for inspiration, but their paths of research were divergent. Unlike Pearson's work, Ball's research was well accepted within the scientific community.

Ball's theory of screws began with a purely physical assessment of a simple mechanical motion.¹⁰⁹ The problem of describing this motion mathematically intrigued Ball, but as he developed the mathematical theory of the motion of a screw the theory took on a life of its own and captivated his imagination. Ball never meant to describe all mechanical motions with his theory, but limited his research to those small oscillations or vibrations that could be described by the generalized screw-like motion. Over the course of years, the theory evolved from the description of a simple motion to the study of a system of screws and the mathematical study of the motion of the system.

Ball's theory was quite well known in England as well as internationally. Many mathematicians contributed to its development in small ways, but it was primarily Ball's theory. In the early years of the 1880's, Arthur Buchheim made some important contributions to the theory.¹¹⁰ He was interested in generalized algebraic and geometric systems and worked directly from Clifford's published work and unpublished fragments. He also corresponded with Ball and Sylvester. It seemed as if Buchheim might be a worthy mathematical successor to Clifford, but, like Clifford, consumption claimed his life before he could reach his full mathematical potential.

In 1897, Ball completed the task that he had originally set for his research.¹¹¹ At first, the simple screws gave way to instantaneous, reciprocal and impulsive screws until Ball could completely describe a system by a screw-chain. His theory was complete when he found the method of finding the instantaneous screws given the corresponding impulsive screws. Ball had taken his theory to its logical limits within the normal Euclidean context.

However, Ball's 1873 discussions with Clifford had convinced him that the theory of screws was "obsolete; it is all going over into non-Euclidean space."¹¹² This early assessment was not completely true; there was still work to be done in completing the Euclidean portion of the theory. Ball did not attempt to study the non-Euclidean aspects of his theory during Clifford's lifetime. It was only after Clifford died that Ball¹¹³ began to develop a non-Euclidean mechanics of vibrational motion.

Ball was not a mathematician, but an astronomer. His intrigue with the non-Euclidean geometry was twofold and extended beyond just the mechanical theory of screws. In his professional duties as the Astronomer Royal of Ireland, Ball made many parallax observations. In 1881, he announced the results of some of these observations before a group at the Royal Institution in London. He concluded his presentation by stating that his observations were not accurate enough to determine between the Euclidean and non-Euclidean nature of space.¹¹⁴ This one statement clearly demonstrates that Ball believed in the reality of a non-Euclidean physical space. In 1885, when Ball wrote the article on "Measurement" for the ninth edition of the *Encyclopaedia Britannica*, he summarized his findings on parallax measurements, but did not commit himself to any particular geometry of space.¹¹⁵ The article was actually an exposition of the latest

advances in the non-Euclidean geometries, further enhancing the recognition of his expertise in this area of study.

During the early 1880's, Ball wrote several articles and memoirs on the non-Euclidean aspects of his theory of screws, but he came to an impasse during the latter part of the decade and returned to the completion of his original Euclidean study of screws. The deeper he journeyed into the non-Euclidean aspects of mechanics, the more difficult it became for him to philosophically justify his work. The problem revolved about the intrinsic, projective interpretation of distance that limited geometry to the Euclidean space. All the terms traditionally associated with geometry carried an Euclidean bias. In 1887, Ball penned his essay "On the Theory of Content"¹¹⁶ in an attempt to come to terms with the resulting philosophical discrepancies in the non-Euclidean geometries. Ball developed a complete new terminology for the study of geometry that he thought devoid of any bias or preconceived notions of space. For example, he no longer referred to the distance between points of space, but the interval between elements in a "content." His hope was to dissociate geometric terms in Euclidean geometry with similar concepts in the non-Euclidean geometry. It is worthwhile to note that he dealt with a "content" of four elements in his derivations, or rather, a four-dimensional space in the biased language of Euclidean geometry. The "content" or space that he dealt with was elliptic,¹¹⁷ just like space in Clifford's theory.

Ball rarely commented on the physical phenomena to which his theory might be applied. However, in an 1885 review entitled "The Theory of Screws," Olaus Henrici implied that Ball's theory would eventually be used to describe the vibrations of molecules and the transmission of vibrations between molecules.¹¹⁸ In other words, the implied physical applications of the theory were the absorption, emission and transmission of electromagnetic vibrations. Ball confirmed this interpretation in his 1887 presidential address before the British Association, "A Dynamical Parable." In this allegorical dialogue between scholars, Ball stated "all instantaneous motions of every molecule in the universe were only a twist about one screw-chain while all other forces of the universe were but a wrench upon another."¹¹⁹ In this statement and following comments, Ball confirmed Clifford's belief and his own opinion that all motion could be reduced to the geometry of position in an elliptical space. In Ball's perspective, the three-dimensional Euclidean analogue to that description would be exhibited by his theory of screws.

After he completed his theory, Ball collected and summarized all of his work in *A Treatise on the Theory of Screws*.¹²⁰ The last chapter of the *Treatise* was an exposition of the non-Euclidean aspects of his theory while his "Dynamical Parable" was included as an appendix to the volume. This book should have been the final chapter of the story on screws, but the final chapter on non-Euclidean geometry was incomplete and opened new vistas for the expansion of Ball's theory. For the past few years, Ball had been working with Charles Jasper Joly in the hope that his screw system could be expanded through the quaternion algebra. Ball was now too old to carry on the task alone and he found in Joly both a willing and able collaborator. But Joly died in 1905 and Ball could do little more to further his theory in the direction of non-Euclidean spaces. He continued work on the

expanded theory nearly until his death in 1913, but the attempt was futile. The association of screws with quaternions in conjunction with advances in physics in unexpected new directions after the turn of the century doomed the theory of screws to an undeserved respite in historical oblivion even though it was popular among mathematicians and scientists at least until Ball's death.

Ball continued Clifford's work along the lines of the dynamics of non-Euclidean space, but Pearson considered the ethereal mechanics that corresponded to the twist as well as the most general philosophical and methodological aspects of science implied by Clifford's ideas on science as an academic discipline. In the view of later scholars, Ball's theory of screws, Pearson's ether squirts and Clifford's *Elements* would all suffer from too close an association with their Victorian counterparts. As science changed, their ideas fell by the wayside. Yet, in each case these scientists were forging new ground in breaking away from the Victorian attitudes with which their work was associated. In the case of Charles H. Hinton, the association with Victorian attitudes neither harmed nor helped. Hinton's early work on hyperspaces and non-Euclidean geometries played to the more spectacular interests of the common public.

Hinton became a student at Oxford in 1871 and was associated with the University until receiving his Masters degree in 1886. Hinton may have had no direct contact with Clifford, but there was ample opportunity for him to come into contact with Clifford's ideas. He studied geometry at Oxford while H.J.S. Smith, who wrote the introduction to Clifford's *Mathematical Papers*, held the chair of Savilian professor of geometry. After Smith's death in 1883, Sylvester was elected to the chair and returned to England from America. This was two years before Hinton finished his work at Oxford. Given the fact that Hinton studied and taught geometry, he would have undoubtedly come into contact with these two men while at Oxford.

However, Hinton's first publication, "What is the Fourth Dimension?" came in 1880,¹²¹ before Sylvester came to Oxford. It was republished with other popular essays and pamphlets that Hinton had written in a book under the title of *Scientific Romances* between 1884 and 1886.¹²² Hinton developed a rough model of a four-dimensional space in his essays, but the greater part of his writing was devoted to the visualization of the fourth dimension by the human mind as well as ethical and metaphysical aspects of the fourth dimension. His writing was aimed at the general reading public rather than scientists and mathematicians. This early work included no mathematical development other than a crude verbal model of space.

The model of space first proposed by Hinton was a three-dimensional sheet of ether in which atoms were embedded. The complete structure was curved within a fourth dimension. The material atoms were likened to threads passing through the sheet from outside the three dimensions of the sheet, the points of intersection representing the individual atoms.¹²³ In this model, Hinton could only account for some of the fundamental properties of matter. In another of his essays, he introduced "twists" as mechanical models of electrical activity.¹²⁴

Although he used terms similar to those used by Clifford and his twists were very like Clifford's, Hinton's twists were non-mathematical visual gimmicks. If Hinton had not been aware of Clifford's work before these essays were published, it would be nearly impossible that no one would have pointed out the similarities between his concepts and Clifford's. Unfortunately, Hinton gave no one credit for his ideas, nor did he make any references or citations of previous work in these early essays. His model of space became more elaborate in succeeding essays. The model evolved into a sheet of ether, curved in the fourth dimension like a sphere upon which material particles followed grooves on their courses through time,¹²⁵ like the needle of a phonograph following the grooves in a record.

Hinton eventually moved to America where he taught at Princeton and other schools before settling in Washington, working at the Naval Observatory. He began this job shortly after Newcomb retired from the post of director of the Observatory and there has been some speculation that Newcomb, knowing of Hinton through their mutual interest in hyperspace theories, secured the job for him.¹²⁶ In 1891, Newcomb offered a brief model of the physical world as a four-dimensional ether. Our three-dimensional space was sandwiched between layers of the ether. The primary purpose of this model was to account for the negative results of the Michelson-Morley experiments in detecting the ether.¹²⁷

In 1891, W.W. Rouse Ball also published a theory that assumed a four-dimensional curved ether.¹²⁸ Rouse Ball's purpose was to explain gravitation and other physical phenomena. While Newcomb's model was only presented before the Washington Philosophical Society, Rouse Ball's theory was published in the *Mathematical Messenger* as well as several editions of his *Mathematical Recreations*, which were published before 1915. In both cases, the models of curved space were quite similar to Hinton's. Rouse Ball discovered Hinton's work after publishing his own theory and pointed out that his theory represented a case of independent discovery.¹²⁹

It cannot be accepted as purely coincidental that those men who developed such models were all associated with Clifford in some manner. Rouse Ball had taken over Clifford's duties at University College when Clifford took his first leave to deal with his illness. It would not be unfair to conclude that these models of space were in some part derivative, either knowingly or unknowingly, from Clifford's theoretical work. In the case of Hinton, especially when considering his use of twists to explain electrical phenomena, the similarities are too remarkable to assume that they were developed in a vacuum.

Hinton's early development was purely philosophical, if not metaphysical, and aimed toward a popular audience. After settling in America, he turned to developing the mathematical aspects of his model. In 1902, he presented a paper before the Washington Philosophical Society, "The Recognition of the Fourth Dimension," in which he finally presented a theory of non-Euclidean space.¹³⁰ The philosophical and explanatory portions of the presentation were republished in *The Fourth Dimension* of 1904, but the mathematical portion of the theory was deleted. Hinton explained that the positive and negative aspects of electricity depended upon the anti-symmetric parts of a Hamiltonian

quaternion.¹³¹ He published a short paper in a mathematical journal indicating the relation of his theory of quaternions to Cayley's work in algebra,¹³² but the complete mathematical theory was never published.

Of all these theories and publications, Ball's was the most popular among serious mathematicians. Pearson's *Grammar* was quite popular, but some of his ideas were met with skepticism when he first published the book in 1892. The third edition of 1911 contained a chapter on the new advances in physics including a summary of recent research on both the electrical theory of matter and relativity.¹³³ In spite of the new advances in science, Pearson chose to leave most of his *Grammar* unaltered, including his long explanation of ether-squirts and the sections on matter and space-curvature. Hinton's earlier books became very popular. They seemed to attract everyone from serious mathematicians to spiritualists and mystics. Hinton did not support spiritualism, nor did he approach the subject in his publications, but some of his speculations could be termed mystical, allowing quite a wide interpretation of his work. In any event, Hinton's 1902 theory remained obscure and apart from his earlier popular publications.

Even while these scientists were working on theoretical models, the popularity of the non-Euclidean geometries and hyperspaces grew rapidly. Ball's and Buchheim's work did little to popularize the concepts, but Pearson, Hinton, Frankland, Halsted and others wrote enough to pique the interest of scientists, scholars and laymen alike who read their publications. During the early 1880's, mathematical expansions of non-Euclidean dynamics were in order. Not only did Ball and Buchheim work in this area, but also major contributions were made by R.S. Heath¹³⁴ and Homersham Cox.¹³⁵ Clifford had founded the study of non-Euclidean dynamics and these men carried on that work after his death. But this early expansion of the study of dynamics was accompanied by a slight lull in philosophical discussion of the concepts as Clifford's ideas were assimilated and evaluated. That evaluation was not all positive. Attacks on the non-Euclidean concepts of space were mounted by Cayley,¹³⁶ Samuel Roberts¹³⁷ and J.B. Stallo.¹³⁸ In spite of this negative reaction to the non-Euclidean geometries, there was an explosion of interest and popular articles on the subject in the latter part of the 1880's and the 1890's.

The geometry that Clifford developed in 1873 was rediscovered and redeveloped by Klein and W. Killing about 1890, inspiring a new look at the physical possibilities of non-Euclidean spaces.¹³⁹ Philosophical discussions of the physical reality of higher dimensions and non-Euclidean spaces became quite common in popular journals as well as professional publications. Perhaps by this time the initial shock of the possibility of non-Euclidean geometries had worn off and scholars became used to the idea, but the discovery by Michelson and Morley that the ether was undetectable cannot be discounted as an incentive to find alternate hypotheses. Newcomb's 1891 suggestion of a non-Euclidean ether model was a direct result of that failure to detect the hypothetical aether.

In 1892, when Poincare's conventionalist philosophy was first presented to an English audience in a translation of his "Non-Euclidean Geometry,"¹⁴⁰ it was a reaction against both the rising tide of popular non-Euclidean heresies and original scientific work on the subject. The scientific community was beginning a period of radical change, which

affected both its methods and attitudes. The change was as much a product of the non-Euclidean geometries as it was of recent scientific discoveries and experimental results. A vast amount of popular literature on the non-Euclidean geometries and hyperspaces was published throughout this period. There are enough direct references to Clifford as well as allusions to his original ideas, that it can be said with certainty that the seeds he had sown did not lay fallow on the ground. His program was beginning to grow and mature, but not in the way that he had planned. His mathematical work had been severed from its philosophical basis. The mathematical system that he was trying to develop withered on the vine with the failure of Ball and Pearson's theoretical work, but the philosophical concepts were carried forward. The imagination of the educated public caught fire on the subject, as had the more disciplined imaginations of some scholars and scientists. This development was not to abate throughout the period up to and including the rise of relativity theory. Any claims that Clifford's ideas died with him or that he had no followers to continue his work are not supported by the historical evidence and thus unfounded.

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