# CHAPTER III

## INHERENT PROBLEMS OF THE FIVE-DIMENSIONAL THEORIES

### 3.1 CRITICISMS OF THE KALUZA TYPE THEORIES

Within the context of what he was trying to accomplish, the unification of electromagnetism and gravitation into a single hyper-dimensional field structure, Kaluza was moderately successful. However, within the context of a mounting tide of criticisms, the success of the quantum theory and the discovery of two new natural forces, the success of Kaluza's theory was short-lived and generally overlooked by the scientific community. The method of unification used by Kaluza, marked by his total dependence upon the basic assumption of a fifth dimension, forms the basis of nearly all of the criticisms of the theory. Kaluza added no physical significance to his fifth dimension, using the concept only as a mathematical tool by which to reach his goal of unification. Yet, his theory was severely criticized both for going too far by even using the fifth dimension, or, on the other hand, by not going far enough and adding some physical significance to his fifth dimension. Even those criticisms that dealt with the fifth variable $\gamma_{00}$, A-cylindricity or the correlations made by Kaluza between the derived mathematical constants within his theoretical structure and known physical constants all depended indirectly on the question of his initial use of a fifth dimension.

Many physicists and philosophers consider any five-dimensional formalism, such as the one used by Kaluza, to be artificial since the world is perceived as purely four-dimensional. Those who criticize such theories find no difference in whether the mathematics are considered real or not and charge all such theories with the pressing need to qualify the five-dimensional assumption ***as if*** the fifth dimension were considered real from the outset. The artificiality with which these theories are charged allows them to appear devoid of any physical content in the fifth coordinate. However, it can be argued that "The success of a language adapted to a five dimensional manifold is, …, only a way of concealing the lack of developments truly adaptable to the four-dimensional universe, which remains the true physical universe."[1] Such arguments may have created an atmosphere within the scientific community whereby the possibility of ever developing a successful five-dimensional theory has been unnecessarily and unfairly hampered. The fact that nearly all of the criticisms hinge upon the existence of the fifth dimension would seem to indicate that objectivity within the scientific community regarding these theories has been lacking and that theories which seem contrary to our psychological interpretations of the objects and events that we sense in the external world should never be seriously considered. That notion is completely unscientific. Any question of the reality of the fifth dimension is crucial to any theory based on any assumption dealing with a five-dimensional component.

Kaluza originally left this question entirely open, but other scientists have since sought to clarify his oversight. Klein, Kaluza's most immediate successor, tried to use the five-dimensional hypothesis to account for quantum effects, as have others since Klein, while some scientists deemed it necessary to either extend or alter Kaluza's five-dimensional space-time structure to meet the challenge of criticism regarding the reality of the fifth dimension. These theories sought to explain why only four dimensions are physically discernible: The projective theories explain away the fifth dimension geometrically, while Einstein, Bergmann and Bargmann's and the Jordan-Thiry theories tried to give some physical significance and content to the fifth dimension. Einstein's final comment on the reality question was given in the second appendix to the fourth edition of *The Meaning of Relativity*. Within this context, Einstein stated that any such theory could be regarded *if and only if* (my own strong qualification) it could be shown why all empirical data leads to a strictly four-dimensional world.[2] The five-dimensional theories should not be forsaken due to this and similar arguments, as long as they can justify their first assumption of a fifth dimension by other means.

Once the fifth dimension has been assumed, any question of the mathematical role played by the fifth dimension in our world naturally evolves into the question of its reality and existence. The question of reality cannot be separated from any purely mathematical consideration of the hyper-dimensional framework or space-time as Kaluza originally intended. The mathematical role of a fifth dimension can be considered in several related, although different ways. Since we have no intuitive or "pretheoretic account of even the qualitative features of a possible fifth dimension,"[3] we have no guidelines by which to consider the fifth-dimensional component, leaving the role that the fifth dimension plays within the theory unclear and open to speculation. This circumstance gives theoreticians a wide latitude to justify such theories in the long run.[4] A crucial factor in our normal space-time is that of the 3+1 division of space-time or rather its mathematical signature of +++-. By giving the variable $\gamma_{00}$ a positive value rather than a negative one, so that bodies always attract each other, (Wolfgang Pauli has shown that the positive factor is related to the gravitational constant and thus attractive in nature),[5] the fifth dimension is space-like rather than time-like.[6]

It has generally been assumed that Kaluza's choice of a negative value would have given the fifth dimension time-like qualities. This choice of sign seems to have been a matter of mathematical expedience in the absence of sound intuitive judgment. A present lack of physical evidence for the existence of a fifth dimension precludes the lack of evidence about whether it is space-like or time-like. Kaluza's theory would seem to indicate the space-like nature of the fifth dimension, yet the case is not closed regarding the signature of the fifth dimension and it could be that it does have a negative sign making it time-like. It should be remembered, "Even if it is in someway space-like, the fifth dimension differs much more from the three ordinary space-like dimensions than does time. We would need some additional conceptual distinctions, besides that of space-like versus time-like, to separate it from the other four."[7]

The role of the cylindrical condition with respect to the fifth component of the field also has a mathematical basis in the absence of intuitive guidelines. The cylindrical condition allows the four dimensions of space-time to exist independent of the fifth dimension, which, in a way serves to explain why there is no physical evidence of a fifth dimension. All observables are four-dimensional and thus independent of the fifth dimension. The cylindrical condition also limits the kind of coordinate transformations possible, allowing only those which lead to covariant field equations,[8] since the fifth coordinate must play a special role which could be space-like in nature. This special role is evident under the cut-transformation by which anti-symmetrical derivatives of the A-curve vanish while the anti-symmetrical derivatives of $A_\mu$ remain allowing a correlation with the magnetic field. The cylindrical condition thus *seems* necessary to the successful unification of the electromagnetic and gravitational fields in Kaluza's theory.

Even though the cylindrical condition *seems* absolutely essential to the Kaluza theory, its imposition has been an important point of criticism of many of the earlier five-dimensional theories and constitutes a major point of weakness in Kaluza's theory. The special status or peculiarity of the fifth component of the field is revealed by this condition. Some scientists have interpreted this condition by as being too restrictive or merely an "additional" condition that is not necessary[9] for precisely this reason. The cylindrical condition was originally used to limit the fifteen possible field variables to the fourteen necessary to describe the combined field. "The condition thus makes it impossible to achieve a complete synthesis in the way that, for example, Maxwell's theory achieved a synthesis of the electric and magnetic fields."[10] It would therefore seem possible that a condition less stringent than the cylindrical condition could be used to derive the same results for the fourteen equations describing a combined field, while leaving the fifteenth equation intact to describe other field phenomena. This alteration would leave the five-dimensional theory more general in its approach.

The perceived weakness of cylindricity has lead to attempts to change or modify the cylindrical condition, rather than justifying its use. In the projective theories, the cylindrical condition is interpreted quite naturally as a projective condition which demonstrates the purely auxiliary role of the five-dimensional space.[11] The cylindrical condition can also be said to lead to a mere codification within a five-dimensional formalism, such that it is a mathematical convenience rather than a physical characteristic of space. In that case, it has been assumed that the five-dimensional space is real. There is a true five-dimensional geometry that can describe space-time, rather than a geometrical (mathematical only) formalism representing space-time, then the cylindrical condition could be modified or dropped altogether. Einstein, Bergmann and Bargmann, as well as Podolanski took this approach to the problem. In their theories, the extra dimension, or dimensions in the case of Podolanski's theory, is considered to be real, but of special structure. In one theory, Einstein, Bergmann and Bargmann used a fifth dimension that was closed with respect to the four dimensions of normal space-time. On the other hand, Podolanski solved the problem by assuming a "laminated structure" such that all the points in a given layer correspond to a given point in the four-dimensional space-time"[12] continuum. To some extent, these theories came as responses to answer those criticisms that attacked Kaluza's dependence on the cylindrical condition.

As stated above, the five-dimensional theories seem to be purely formal in that they represent mathematical formalisms independent of any reality given the fifth dimension. They seem non-intuitive, in that we have no previous notion of a five-dimensional space-time curvature, and ad hoc.[13] These theories also stand accused of being merely synthetic and non-predictive (except for possibly the Jordan-Thiry approach),[14] in that Kaluza's and many other theories do not expand upon the Einstein-Maxwell equations. In its design and conditions, the Kaluza theory only reproduces the Einstein-Maxwell equations within the structure of the combined field that Kaluza developed, and various interpretations of unidentified mathematical constants have been made for the theory to match the Einstein-Maxwell equations exactly. Some interpretations, which may not have been completely justified in a scientific manner, except through hindsight, were made to exactly correlate the results of the Kaluza theory with the electrodynamic equations. These occurred with the identification of the scalar quantity $^2/2$ in the final equation describing the field with the gravitational constant k of General Relativity and the identification of the field vectors $A_n$ (= $\gamma_{0i}$) with the electromagnetic potentials $\phi_i$. These identifications were made simply because of the commonly shared characteristics between mathematical constants and their physical counterparts. There were no other sound scientific justifications for either of these identifications, but likewise, there were no scientific reasons why they could not be made. So, it has been argued that electromagnetism was not incorporated into Kaluza's field structure in a natural way[15] as he originally desired. It has also been stated that the identification of the $A_n$ with the potential $\phi_i$ does not actually establish a geometrical character for the electromagnetic field[16] such that it has not been demonstrated that "*only* Maxwell's equations could be combined with Einstein's into a single formalism."[17] Pauli went so far as to say that Kaluza's representation "is in no way a 'unification' of the electromagnetic and gravitational fields. On the contrary, every theory which is generally covariant and gauge-invariant can also be formulated in Kaluza's form."[18] In other words, it was claimed that the Kaluza theory did not actually depict a single field from which both electromagnetism and gravitation could be exclusively derived.

In spite of such criticisms, it still remains true that the geodesics of charged particles were derived within a combined field, which seemed to unify the two basic fields into one combined field structure. However, the field equations of the combined field were themselves derived from a variational principle in which the Lagrangian still appeared as the sum of the two terms;

$$L = (-g)^{1/2} [R+(k/2)F_{ik}F^{ik}] .$$ [19]

Here, R is again the scalar representing space curvature and $F_{ik}$ (or $F^{ik}$) is the electromagnetic field strength. Even though the field has been unified into a single structure, there is still no single tensor that can be used to represent both the electromagnetic and gravitational components of the field. The problem of adding the electromagnetic tensor to Einstein's original field equation from outside the theory could thus seem to have been transferred from to the introducing the Lagrangian in the five-dimensional theory to some scientists. This fact left the theory with a synthetic appearance without completely establishing the geometric character of the electromagnetic field within the combined field. So the purpose for which the search for a unified field theory had begun seemed not to have been completely satisfied to some critics of the theory.

## 3.2 A REBUTTAL OF SOME CRITICISMS

The Kaluza theory is successful in its own way and can be seen as an indication of one direction that could (or should) be taken in further theoretical research. However, very few scientists have followed this research path or otherwise reacted to the implications of the theory, a fact that underscores the *limited* success of Kaluza's theory. Some of the most damaging criticisms claim that the five-dimensional theories are nonpredictive, artificial, synthetic, ad hoc, merely codifications and have no pre-theoretic or philosophical basis. These criticisms understate the success of the Kaluza theory as well as its simplicity. The fact that the theory *at the very least* duplicates the accepted results of Einstein and Maxwell should indicate to scientists that the possibility of extending this theory is a viable alternative to other forms of unification.

Only if Kaluza's theory is seen as an endpoint in itself, a finalized theory without possible extension, can the previous criticisms be considered absolutely valid. But, if the criticisms were used constructively to pinpoint shortcomings within the theory, with the express purpose of alleviating those shortcomings, then the criticisms would help in the overall advance of science. The criticisms have provided an important impetus for the extensions, but these extensions have not been well received, just as Kaluza's original theory was not that well received. The fact that the attempted extensions of the theory have met with little support in the scientific community (this is true of all unified field theories) would seem to attest to both the lack of support for the basic hypothesis of a hyper-dimensional space-time structure as well as the strength of the present paradigm of physics, the quantum mechanical approach to nature. The present paradigms are valid, but the quantum approach to reality is not absolute to the point of excluding and invalidating other theories and worldviews, which may or may not fall outside of its scope. Any field theory will eventually have to cope with the successes of the quantum theory, just as quantum mechanics will eventually have to cope with the successes of the field theories. Until a new synthesis of discrete and continuous is accomplished to the overall satisfaction of science, Kaluza's theory offers a valid method as a possible alternative to complete the task.

The general non-predictive nature of the theories represents a major difficulty with the five-dimensional approach. This problem stems from their artificial and synthetic formalism as well as their lack of a theoretical philosophy. But the criticisms regarding the non-predictive nature of these theories is not as completely valid as one might be made to believe. These criticisms only take into account the main-line theories, of which the Jordan-Thiry model alone is predictive. When theories outside of the main-line of development are included, several theories predict a minimum length and/or time,[20] Corben's extension of Special Relativity predicts gravitational phenomena which are as yet unknown,[21] and Flint has used his theory, at an early stage of its development, to predict the stability of the atomic nuclei.[22] So the five-dimensional theories, as a group, cannot be criticized as non-predictive, since this criticism refers to only one group of the theories. However, it is true that the predictability of the five-dimensional theories is severely limited, as was the predictive power of General Relativity when it was first developed. Perhaps by focusing on a philosophical, as well as mathematical approach, the predictive power of these theories could be increased.

The generally accepted assumption that the extra dimension is space-like, rather than temporal in nature, has also been criticized. This argument results from the use of a positive sign for the value of $\gamma_{00}$, the positive sign being clearly associated with the space-like character of gravitational attraction. Yet it would be difficult, if not wholly impossible, to guarantee that making any value of $\gamma_{00}$ positive renders the fifth dimension space-like without any reservations or qualifications. There is a greater deal of difference between the space-like and time-like characteristics of our world's structure, than the mere change of a mathematical signature could possibly indicate. The positive and negative *mathematical* signs could well be inconsequential in the face of the overwhelming qualitative *physical* differences between space and time. Perhaps the analogies between the qualitative *physical* characteristics and the choice of sign are merely coincidental. If mathematicians and physicists are so sure that a positive sign has no other *physical* function than demonstrating the space-like character of attraction, it would be simple to assume that the opposite sign, given to time, would mean that time is repulsive. This notion would be wrong, as it would be equally wrong to assume that either a positive or negative value for $\gamma_{00}$ would make it either space-like or time-like. Graves admitted that "the fifth dimension differs much more from the three ordinary space-like dimensions than does time,"[23] a fact which should be clear to anyone. We have no reason to assume that the fifth dimension is either space-like or time-like even though we only have a mathematical choice of two signs to represent the fifth dimension. Whichever sign we choose to represent the fifth coordinate, it may merely indicate a mathematical quirk rather than a physical characteristic.

The mere assumption of a five-dimensional space-time structure can take either of two forms, both of which are not as independent of the other as one would expect or hope. First of all, the five-dimensional structure may be used as a mathematical formalism that is only meant to allow for the extra variables corresponding to the degrees of freedom needed to incorporate the electromagnetic and gravitational fields into a single geometrical framework. In this case, the fifth dimension is assumed to have no physical meaning. Kaluza's theory assumed this particular form. Otherwise, the fifth dimension could be given an actual physical meaning, with the properties of the fifth dimension depending on the theoretical structure with which any particular theoretician is working at any given time. It is necessary that every theoretician explain why the fifth dimension seems to be beyond experience and justifies his or her choice of properties for the fifth dimension.

The assumption of a fifth dimension in either of the above forms evokes a number of criticisms that do not distinguish between the two different forms, although they should. It would seem that the different criticisms would be specific to one form or the other, but this is not the case. The Kaluza theory is criticized for utilizing mathematical assumptions that serve only to reproduce the necessary equations (Maxwell's and Einstein's) without any physical basis. One form that this assumption takes in Kaluza's theory is that of a value of one for $\gamma_{00}$. This value is critically associated with the condition of cylindricity. Yet, in the case where the Kaluza theory is meant to be only a mathematical formalism, the assumed value of $\gamma_{00}$ and the cylindricity condition are both criticized for being assumptions which have no basis in physical reality, even while five-dimensional theories in general are criticized by way of the fact that they have no pre-theoretic or intuitive notions with which to conceptualize the fifth dimension.

A double standard seems to have been applied in these criticisms, which is partly the fault of the theoreticians themselves. In Kaluza's theory the condition of cylindricity and the non-dependence of the four coordinates of space-time on the fifth dimension are used to explain why the fifth dimension is never experienced or observed. However, this explanation opens the door for critics to claim that Kaluza utilized a real fifth dimension instead of a mere mathematical formalism in his application of the five-dimensional concept. There is no basic difference between these two forms of theories as far as the criticisms are concerned. Either the theoretician assumes the reality of a fifth dimension or his mathematical equations and formulations imply its reality, so the opponents and critics of the concept can criticize either type of theory on this same account. The theoreticians could never escape the eventuality of assuming that their concepts are related to real worldviews, no matter how hard they tried to adhere to purely mathematical models devoid of any physical reality.

Once a fifth dimension is assumed in any way, means, shape or form, the theory is not allowed to remain purely mathematical by the critics. The fifth dimension is required to be physically real as well. In this sense, if the fifth dimension has been introduced merely as a mathematical tool and meets with any success, even the smallest success, the reality of the fifth dimension would be strongly implied and questions concerning the reality of the fifth dimension would surely be considered: Why isn't space-time five-dimensional as is the mathematical framework that successfully describes it? On the other hand, if the real space-time structure is expected to be five-dimensional, why isn't the fifth dimension experienced?

It would seem that the mere consideration of a fifth dimension raises profound philosophical and scientific questions regarding the reality of the fifth dimension and our perception of a three-dimensional space with a single time dimension. Perhaps this is why questions regarding the dimensionality of space became so popular during the last century, well before the Riemannian geometry was given a successful physical basis in General Relativity. The question of dimensionality was later recognized by Einstein in his statement that any hyper-dimensional theory must account for the *apparent* restriction of experience to the four dimensions of space-time.[24] This particular problem introduces a maddening circle of dependence for the various criticisms of the five-dimensional theories, concerning both forms of assumptions of a fifth dimension. Therefore, it is quite evident that there is only one true criticism of the hyper-dimensional theories, the adoption of the fifth dimension itself.

Perhaps the method of immediately pursuing mathematical extensions to Kaluza's theory is the wrong approach to the five-dimensional dilemma. If the only true criticism of these theories concerns the adoption of the five-dimensional concept itself, then more emphasis should be placed on the philosophical implications of a possible fifth dimension to act as a guide in applying the mathematics. A direct assumption of the reality of the fifth dimension could and should be made as the basis of any new theory. Upon this assumption, a new philosophy could be developed to act as a guide to any qualitative and quantitative speculations of how simple mechanical and electromagnetic phenomena could be explained. Questions could be raised and answered along this line of reasoning in order to build the pre-theoretic or intuitive picture which critics have claimed lacking in previous five-dimensional theories. Such questions might take the following form: How would Lorentz contraction appear in a five-dimensional space-time continuum? Since mass changes according to Special Relativity with increasing velocity and mass is related to curvature, how would a changing mass appear in a five-dimensional structure? How can electromagnetic fields fit into a five-dimensional scheme of space-time? Why, or how, does the four-dimensional continuum appear to curve in the five-dimensional worldview? Given no intuitive background concerning the fifth dimension, a philosophical structure built from such questions would seem to be in order. This philosophical structure could render the concept of a fifth dimension far more natural and intelligible with respect to our present knowledge of physical phenomena. No one has yet attempted to construct such a philosophy, or at least there are no references to one. Even so, it should be acknowledged that such a philosophy is needed. Among others, Peter Bergmann has noted that hyper-dimensional theories lack a "convincing and complete physical interpretation."[25] A philosophical structure as described above could fill this gaping hole in the theoretical model of a physical space-time of five dimensions.

## 3.3 CURVATURE, DIMENSION AND JUSTIFICATION

### 3.3.1 Higher Dimensions

In the above historical development, a number of themes have been proposed which now need more elaboration. These include statements that the application of Riemannian geometries somehow implies the actual physical existence of spaces of higher dimensions and that there exists a predilection to maintain a 3+1 space with time (or to a lesser extent a four-dimensional space-time) structure despite some indications that these concepts may need revision. These claim are evident in some of the criticisms of the five-dimensional theories, and alone, they are enough to discourage most speculation on the hyper-dimensional space-time structures, perhaps unnecessarily. It is not denied that any five-dimensional theory must, as Einstein so aptly stated, "explain why the continuum is *apparently* restricted to four dimensions."[26] However, in the light of the successes of some of the five-dimensional theories, some criticisms are unjustified and do not warrant the unpopularity of nor the negative attitude toward the theories which these criticisms helped to foster. It is difficult to agree with Bergmann's statement that such theories are "marred by the absence of a compelling logical necessity."[27] To the contrary, their necessity is quite evident as a compelling operational necessity for their development. Their development has been marred, in turn, by a negative attitude toward these theories that necessitates a far greater justification of their plausibility and viability as legitimate physical theories than would be necessary for other new theories.

Until the first development of Non-Euclidean geometries, the world was described by a three-dimensional space with a single-dimensional time, without argument or challenge. The various non-Euclidean geometries, and in particular the Riemannian geometry, began a period of introspection during the latter nineteenth century in which the questions were raised as to the number of dimensions needed to describe the physical world. Philosophical arguments justifying the three-dimensionality our world would have been neither necessary nor even raised if not for the development of the non-Euclidean geometries. Certainly, there was no physical basis for arguing that space could have more than three dimensions except for the prior development of the new geometries, unless it was through the foresight of some scientists and philosophers concerning coming events in science. Only William Kingdon Clifford seems to have clearly anticipated future developments. The Riemannian geometry merely established a mathematical precedent for higher dimensions and only in this manner did the Riemannian geometries indirectly imply that physical space might have higher dimensions. Clifford directly stated his belief of the possibility of a higher dimensioned physical space and tried to develop a theory of matter based upon that assumption.

The basic problem of the existence of higher-dimensional spaces did not become a viable scientific issue until the advent of General Relativity. By basing a successful physical theory on Riemannian geometry with its concept of curvature, new philosophical and physical standards regarding dimension became necessary. With the success of General Relativity, scientists and scholars were finally forced to contend with questions regarding the physical curvature of space-time. Before the acceptance of General Relativity, a few scholars, as the case may be, had only suspected the curvature of space or space-time, so it was not a viable option for scientists to consider. The initial success of General Relativity to accurately describe gravitational anomalies such as Mercury's advancing perihelion and the bending of light rays around massive objects, curvature became a viable option and the necessity to distinguish between a four-dimensional space-time continuum characterized by intrinsic curvature and one embedded in a fifth dimension exhibiting extrinsic curvature became clear. In the absence of any perceptual or experimental evidence of a higher dimension, scientists settled on the concept of intrinsic curvature to represent physical reality. Intrinsic curvature won the non-debate in science by default, thus science avoided a far more serious dilemma then had been represented by the seeming lack of philosophical discussion upon the topic. Before General Relativity, the more natural concept of extrinsic curvature was always associated with any possibility of our space being non-Euclidean in a final analysis and after General Relativity the concept of extrinsic curvature all but disappeared. This historical anomaly has been neither documented nor explained although an explanation is not hard to find in the words of scientists of the time.

> **Accordingly, if the objective triple space of the universe is actually found to possess the quality of curvature, whenever and in whatever way that quality may be revealed by measurement, the mathematical conception of that curvature would exact the existence of some further space of ultimate reference. That further space**

> **would be more extensive in dimensional range than the three-dimensional objective space of the universe; and, in mathematical conception, it would be characterized by complete linearity.[28]**

In the above statement, Forsythe, a mathematician and not a physicist, does not mean to imply an extrinsic space of higher dimension, but instead would find an intrinsic space of higher dimension perfectly acceptable. Within this statement, the dilemma can be seen to have reached a new level whereby we can take Forsythe's advice and seek curvature in higher dimensions that are intrinsic to the space-time structure or seek curvature in higher-dimensional embedding spaces. The concept of dimension has thus been rendered obscure.

In General Relativity our world is described by Einstein's equation,

$$R_{ik} - \tfrac{1}{2} g_{ik}R = kT_{ik} .$$

It is important to note that all of the terms affecting the curvature of space-time appear on the left-hand side of the above equation, while the lone term representing the presence of matter appears on the opposite side. This creates a balance or equality between the amount of matter and the curvature which can be interpreted in three different ways:[29]

(1) The reality of matter is assumed; such that the curvature is a product of the presence of matter, matter being the *only* reality. This attitude follows closely the Newtonian viewpoint except, of course, for the introduction of the concept of a mathematical curvature.

(2) The reality of matter and curvature are equally emphasized in such a way that they are both physical features of the world. This view is probably more widely accepted today: Matter curves space-time and space-time curvature moves matter.

(3) Curvature is the true reality and matter is only a manifestation of the curvature. This is the more radical view even though Clifford originally suggested it in 1870. It departs completely from the Newtonian concept of matter and has only become a realistic point of view since the EIH (Einstein, Infeld and Hoffman) formulation of motion due to space-time curvature. The EIH formulation represents the beginning of Geometrodynamics as popularized by J.A. Wheeler.

Some modern scientists, such as Erwin Schrödinger, supported the last viewpoint.

> **It is sometimes said that matter determines the curvature of space-time. But the most advisable is, I think, to reserve expression of <u>matter</u> to indicate the object of our direct observation and to regard <u>curved space</u>-<u>time</u> as the picture or <u>model</u> we form of this object in our minds .... [30]**

Schrödinger was saying that matter is observed or perceived, but matter need not represent the final reality. In an even more radical statement, Wheeler took the argument to a still higher level.

> **Is the physical space in which we live a purely mathematical construct? Put the question in another way: IS space-time only an arena within which fields and particles move about as 'physical' and 'foreign' entities? Or is the four-dimensional continuum all there is? Is curved empty geometry kind of a magic building material out of which everything in the physical world is made?: (1) Slow curvature is one region of space describes a gravitational field; (2) A rippled geometry with a different type of curvature somewhere else describes an electromagnetic field; (3) A knotted-up region of high curvature describes a concentration of charge and mass energy that moves like a particle? Are fields and particles foreign entities immersed in geometry, or are they nothing but geometry?"[31]**

Both of these descriptions exemplify the possibility that curvature is the primary element of physical reality, rather than matter itself.

In dealing with matter alone, we at least have a basic notion of what we are dealing with, but this same statement cannot be applied to space-time curvature. The term curvature itself represents a possible misconception that the surface on which the curve exists is always embedded in a surface of higher dimension. The example used in most cases involves the conception of a two-dimensional surface embedded in a three-dimensional space, such as a presented by a simple globe or sphere. But in the case of curved space-time, can we really say what space-time is curved into? Is this even a valid question? According to General Relativity space is curved, but the theory makes no distinction between an intrinsic curvature and an extrinsic curvature, which necessitates higher dimensions. The "manifold in question is clearly non-Euclidean or non-Minkowskian, one might claim this is not sufficient to establish real curvature, unless the term is stretched far beyond its normal meaning."[32] In this sense, curvature is the wrong term in that it conveys a misguided mental picture by which our conceptual model of curvature in three-dimensional space is extrapolated to a fourth dimension. This does not mean that space-time cannot be curved in some sense similar to our mental picture (in a higher-dimensional embedding space). Rather, it means that a higher embedding dimension is unnecessary to fully understand the mathematical concept of curvature. In the language of mathematics and thus science, it is possible to have intrinsic curvature in a four-dimensional space-time without reference to a higher-dimensional embedding structure, such as would be implied by the case of an extrinsic curvature. However, by the same argument, the possibility of a higher-dimensional embedding structure cannot be ruled out for strictly mathematical reasons.

In any theories proposing space-times characterized by more than our normally sensed four dimensions, the reality of our space-time's intrinsic or extrinsic curvature becomes important. Science has largely ignored the latter possibility that curvature implies an embedding space of higher dimensions even though the mathematics is the same in either case. Whether these theories represent the real model of space-time or only discrepancies in measurement, as explained by a mathematical formalism, is another problem dealing with our present inability to find any empirical test for the true nature of the curvature. Simply speaking, intrinsic curvature is that curvature of space-time which can be measured without need to formulate higher-dimensional embedding space, while extrinsic curvature can only be explained in regard to a higher-dimensional space (or space-time) structure. Lawrence Sklar pointed out "Insofar as General Relativity asks us to contemplate a world in which space-time is curved, it is *intrinsic* curvature only that is being discussed."[33] This is not a complete denial of higher-dimensional embedding spaces, but shows that there is no compelling logical necessity within General Relativity to assume a space-time with a number of dimensions greater than four. Likewise, it does not prohibit speculation about such spaces or space-times in the event that the five-dimensional theories can uniquely explain phenomena that are already known to exist or new experimental evidence is found which points to an extrinsic curvature, giving science a very compelling operational necessity for such theories. Until this time comes, if indeed such a situation ever develops, the intrinsic or extrinsic character of space-time curvature can only be regarded as a philosophical question, personal choice or personal bias based upon our psychologically oriented Newtonian principles and our limited perception of the world.

The philosophical question of space-time curvature has been further eroded by the introduction of new elements into the controversy. Adolph Grünbaum[34] argues for the intrinsic character of the metric instead of the curvature, but the metric is intimately associated with the curvature, there is little difference in the end. His argument is based on the 'metrical amorphousness of space,' which is to say that when a continuous, homogeneous manifold is considered, as in the Riemannian space-time, the congruence or incongruence of any intervals cannot be intrinsic properties of the intervals. Hans Reichenbach,[35] an advocate of extrinsicality, has developed another idea in his 'relativity of geometry.' He claimed that alternative geometries can be used to represent the same physical space if they are both based upon valid definitions of congruency. Other geometries can thus be considered as valid, but only as long as one particular geometrical structure cannot be proven to uniquely fit the physical circumstances of our reality. Clark Glymour[36] introduced such an alternative geometry based on a type (1,3) curvature tensor using Grünbaum's ideas, which does not present a metric, and therefore (he claims) has found space-time curvature to be intrinsic. Glymour claimed that space-time curvature must be intrinsic for all geometries according to Reichenbach's 'relativity of geometry.' Wesley Salmon[37] has taken the opposing view and argued against Glymour's hypothesis. The question must ultimately arise, which geometry *uniquely* describes the condition of space-time curvature? The ultimate choice of any geometry used to represent the space-time continuum must rely solely upon physical evidence of such a kind that it leaves no question as to which geometry is valid. At the present time this may seem an impossibility, so philosophical arguments such as these are important to an extent, but are presently doomed to failure in the face of empirical evidence that proves one view over the others.

Until overwhelming physical evidence of the intrinsic or extrinsic nature of curvature is presented, theories of higher dimensions can be proven neither true nor false. Given the present state of affairs in physics, the higher-dimensional hypothesis is not forced on physics by necessity so the choice for supporting or criticizing five-dimensional theories is merely one of personal preference. H. Robertson, recognizing this fact, stated that

> **The answer to this methodological question will depend largely in the universality of the geometry thus found - whether the geometry found in one situation or field of**

> **physical discourse may consistently be extended to others 0 - and in the end partly on the predilection of the individual or of his colleagues or of his time.[38]**

In spite of such open-minded beliefs, there still remains an extremist element marked by the fact that personal preference has surmounted the pedestal of personal bias. Henri Poincaré berated and belittled early attempts to measure the radius of curvature of space by studying the parallax of nearby stars. His view represents an early precursor to that group of post-relativity scientists and scholars who would reject the very validity of hyper-dimensional theories from the outset. As he stated,

> **If therefore parallaxes were found, or if it were demonstrated that all parallaxes are superior to a certain limit, two courses would be open to us; we might either renounce Euclidean geometry, or else modify laws of optics and suppose that light does not travel rigorously in a straight line.**
> **It is needless to say that all the world would regard the latter solution as the more advantageous.**
> **The Euclidean geometry has, therefore, nothing to fear from fresh experiments.[39]**

Even Mach did not go this far in his criticisms of using a non-Euclidean geometry for physical or pseudo-physical purposes. Mach at least admitted that the question of a four-dimensional space (without time) would become a serious one, if it were shown that some phenomena exhibited the four-dimensionality, although the intent of his comment to this effect was an attempt at sarcasm.[40] Another sentiment, similar in a strange way to Poincaré's statement, has been given a theoretical form in the five-dimensional hypothesis of Bennett, Brown and Thring. Poincaré would never accept their assumption of a fifth dimension, but they have done just as he said by interpreting the appearance of curvature to a property of the observer's measurement, rather than to a property of space or space-time. In a way, this theory very nearly changes optics in such a way that the observer's measurement would show the light beam to have curved. Another notion similar to Poincaré's is explicit in a statement by Forsythe.

> **Contemplative minds often attain intellectual satisfaction when they discern correspondence, between their observations of an external world which they call real, and the results of logical theory which they call abstract in relation to such observations. An occasional tendency to interchange the real and the abstract in such correspondences, as though they are equivalent, can even prove obnoxious to lucidity of the statements in which reasoned thought is expressed. One consequence is not rare: confusion is caused in the presentation of a new theory, launched in the name of science. An obvious illustration is provided in the notion of a fourth dimension. The notion was propounded by the mathematicians: The added dimension, which they have incorporated in an abstract geometry, is coordinate in quality and possibilities with the three dimensions familiar already in conceptions of triple space. The fourth dimension has been appropriated by some physicists, for what is called a 'natural' geometry without any requirement as to coordination in quality and in possibilities with the three dimensions familiar to experience.[41]**

Although this statement is not exactly equivalent to Poincaré's, Forsythe does seem to castigate physicists in general for misusing the hyper-dimensional geometries at the expense of our normally sensed space of three dimensions. The attitude that he expressed in this phrase certainly seems to indicate that the strictness of logical thought in mathematics cannot be so easily applied by physicists to a real world situation.

The extremist attitudes that are noted above have not gone without comment by Einstein, in particular with regard to the opinion stated by Poincaré. Einstein accepted the role of experience in physical hypotheses with all of its implications, but did not accept experience as the sole basis for the deduction of mechanics, optics and other physical laws as Poincaré would have done. Einstein was a physicist doing mathematics while Poincaré was a mathematician doing physics. In fact, Einstein directly criticized the attitude expressed in Poincaré's statement.

> **Why is the equivalence of the practically rigid body and the body of geometry - which suggests itself so readily - rejected by Poincaré and other investigators? Simply because under closer inspection the real solid bodies in nature are not rigid, because their geometrical behavior, that is, their possibilities of relative disposition, depend on temperature, external forces, etc. Thus the original, immediate relation between geometry and physical reality appears destroyed, and we feel impelled toward the following more general view, which characterizes Poincaré's standpoint. Geometry (G) predicates nothing about the behavior of real things, but only geometry together with the totality (P) of physical laws can do so. Using symbols, we may say that only the sum of (G) + (P) is subject to experimental verification.[42]**

Also,

> **Against Poincaré's suggestion it is to be pointed out that what really matters is not merely the greatest simplicity of the geometry alone, but rather the greatest possible simplicity of all the physics (inclusive of geometry). This is what is, in the first instance, involved in the fact that today we must decline as unsuitable the suggestion to adhere to Euclidean geometry.[43]**

In other words, it is not the simplicity of geometry alone to which science should bow as Poincaré stated, but the simplicity of geometry and experience of nature taken together. Einstein would have science consider both geometry and experience, without either having absolute priority over the other, to derive the simplest overall view of the physical world. Experience can guide geometry and epistemology must be satisfied, but there is no preordained or 'a priori' geometry to which experience must always adhere. If no geometry can be considered as 'a priori' to a physical theory, as Poincaré considered Euclidean geometry, then the possibility exists that a hyper-dimensional geometry could offer the simplest solution to physical problems. In that case, such a geometry must be given serious attention by scientists.

Possible biases against the five-dimensional theories are limited to neither the non-scientific community nor that part of the scientific community refusing to even consider these theories. Two of the hyper-dimensional theorists have shown discontent with the five-dimensional formalisms in general, but have still opted to use such formulations in their own ways. The Bennett, Brown and Thring theory,[44] in what appeared to be an attempt to save Newtonian mechanism, was based on a five-dimensional concept dealing with absolute straight lines as an extension of Newton's first law, thus denying any possibility of space-time curvature. This development presents a paradox since Newtonian theory, which is intimately associated with the flat three-dimensional geometry of Euclid, could never be compatible with a concept such as a five-dimensional space-time, which is usually considered non-Euclidean. On the other hand, Podolanski, who formulated a six-dimensional theory, considered the five-dimensional theories of his predecessors to be "Ambiguous ... the formulation of the five-dimensional laws of nature becomes a toilsome and unpleasant task. The heuristic value of such a theory is nil."[45] After he made this comment, Podolanski was forced to concede that "still, there are reasons to believe that a hyper-dimensional description of nature is useful."[46] The five-dimensional theories offer specific advantages and Podolanski was forced to backtrack from his harsh opinion of the theories, possibly because he could not escape the realization that these advantages could be gained by any other method.

Theorists attempting to make use of these advantages have had to proceed with caution in the face of skeptical opposition to the five-dimensional theories. Einstein's opinions regarding the five-dimensional theories have already been noted, while Flint, who had done an enormous amount of work in the application of the five-dimensional concept to quantum theory, on occasion felt compelled to comment on the justification of his approach.

> **There is no real objection to the use of such symbolic methods in the attack on problems in physics and the five-dimensional method may be regarded as symbolic, but it was the advantage that it follows well known lines and that well known geometrical terms can be applied to it.[47]**

And,

> **Our use of the fifth coordinate here is solely to enable us to make an appeal to mathematical form. If we are correct in the sense that our results correspond accurately with those of experiment, we can never assume to say more than that the physical world can be described as if it were five-dimensional. The success of the four-dimensional theory of the universe allows us to say no more than this with regard to the four-dimensional world. One may only exclaim in the excitement of discovery that space and time have henceforth vanished to shadows.[48]**

Both of these statements show Flint's theories to be no more than mathematical formalisms, but it is hard to believe that Flint's theories are no more than mathematical in content. Too much physical content had been put into Flint's theories to accept their absolute mathematical and symbolic character.

Only one theorist, in contrast to all others, seems to have taken a definite stand on the issue of accepting five-dimensional theories and supported them without equivocation. Wilson stated, rather explicitly, his support for such theories. "Notwithstanding its attractiveness, the opinion may be stated here that Kaluza's theory is the correct one, though this does not command universal assent."[49] The total lack of support by the scientists working on these theories, ranging in opinion from Podolanski to Flint, again shows the adverse affect that more popular opinion has on the outcome of scientific endeavors. While developing five-dimensional theories, these scientists carefully 'hedged their bets' by publicly stating they were merely playing mathematical games with the concept. How they felt in private, out of the light of criticism, may have been quite different.

The shear inertia of science to adopt new and radical ideas such as space-time curvature may also have contributed to the lack of support for the early development more conservative and accepted forms of General Relativity. According to Trautman

> **In spite of its profound implications, for a long time Einstein's theory was being developed with little contact with the natural sciences. The situation has changed during the last years, thanks to startling discoveries in astronomy, the progress in radio and radar measurements and the patient efforts to detect gravitational waves. The theorists have followed suit and done relevant work on the process of collapse and formation of black holes, on new general relativistic effects, on the mechanisms of emission and absorption of gravitational radiation, and on the stability of relativistic, gravitating systems.[50]**

It could easily be stated that the lack of development of the General Theory during the period from 1915 to 1960 was due to both the overwhelming presence and impact of the Quantum Theory on the scientific community and the lack of a physical ability to make experimental verifications of gravitationally related phenomena. However, a potent argument could also be made for the lack of development due to the overwhelming nature of the revolutionary concepts which General Relativity presented in its ideas on space, time, space-time and curvature, while the development and introduction of experimental equipment fit to the task of supplying empirical support for General Relativity by the 1960s merely provided such overwhelming evidence that the unfavorable and/or questionable theoretical aspects of General Relativity could no longer be ignored. From nearly the beginning, there was too much physical evidence in Einstein's explanation of the advance of Mercury's perihelion and the bending of light rays to deny the theory outright, but the theory remained underdeveloped until its development was begun anew by the scientific community in the 1960s. This, of course, has a direct bearing on the acceptance of the five-dimensional theories since they assume the validity of General Relativity as their starting point for developing a more general unified field theory.

### 3.3.2 The Inherent Problem of General Relativity

The aspect of caution in dealing with the radical nature of the five-dimensional theories also extends backwards to General Relativity, although not nearly to the extent that it is evident in the five-dimensional case. Even as science cannot venture too far in its application of a five-dimensional hypotheses, as with unified field theories in general, there seems to be a limit to how far science may go in dealing with General Relativity. General Relativity cannot be fully accepted as long as the space-time curvature is unproven, or misunderstood, as a real physical characteristic of our world. Direct proof of physical curvature can only be found by extrinsic measurements and in their absence the reality of intrinsic curvature is only implied by the success of General Relativity to explain known phenomena and predict new phenomena. The extrinsic character of curvature, in turn, permits the possibility of hyper-dimensional space-times. Einstein, although not cautious himself regarding the limits to which General Relativity could be carried, reacted to the lack of acceptance for the notion of curvature in the statement of his view of the nature of physical reality. Not willing to accept geometry at the expense of experience, or experience at the expense of geometry, he professed the opinion that only the simplest framework, taking both experience and geometry into account, could successfully explain the physical world.

> **Our experience hitherto justifies us in believing that nature is the realization of the simplest conceivable mathematical ideas. I am convinced that we can discover by means of purely mathematical constructions the concepts and laws connecting then with each other, which furnish the key to understanding of natural phenomena. Experience may suggest the appropriate mathematical concepts, but they most certainly cannot be deduced from it. Experience remains, of course, the sole criterion of the physical utility of a mathematical construction. But the creative principle resides in mathematics.**[51]

Within such a worldview of physical ideas, the curvature of the continuum depends on the choice of geometry used to describe the continuum. Where experience is incapable of guiding the way to a better understanding of the concept of curvature, geometry must lead the way. In Einstein's point of view, curvature emerges as the only simple way to explain gravitation. Caution then becomes unnecessary in advocating the use and extension of General Relativity. However, this view is not universally accepted and only the strong evidence, which supports General Relativity, has forced the acceptance of Einstein's view of physical reality on a community that would rather find another more palatable explanation of gravity, matter, space and time. Evidence that the scientific community would rather find another explanation for gravity than the curvature of space-time can easily be found in the attempts to quantize gravity and thus do away with the concept of the continuous field.

Bergmann has done a great deal of work on both General Relativity and the five-dimensional concept, but he is still capable of regarding General Relativity with some caution.

> **The General Theory of Relativity is Einstein's theory of gravitation, right now the only theory of gravitation that, after Newton's, has achieved a measure of universal recognition.**[52]

It is interesting to note that Bergmann did not assume that General Relativity is an accomplished fact of reality in making this comment and has only rated it as having a *measure* of universal recognition, despite the fact that no other gravitational theory had been able to account for the experimental evidence that supports General Relativity. In referring to five-dimensional theories, Bergmann has stated that

> **... at present, however, all such theories - as unified field theories generally - lack a convincing and complete physical interpretation. In this way they are little better off than Einstein's theory of gravitation itself. But unlike the theory of gravitation, they are also marred by the absence of a compelling logical necessity.**[53]

Thus, Bergmann seems quite willing to show caution in his complete and public acceptance of General Relativity (or he may just be cautioning others to take caution regarding it), and in so doing states that Einstein's theory lacks a "*convincing and* *complete physical interpretation*" only to a slightly lesser extent than the five-dimensional theories themselves. Bergmann, at least within this context, could have been referring to either the concept of curvature itself, the ambiguity in the interpretation of curvature or the five-dimensional theories dependence on the concept of curvature.

It is the above stated lack of a physical interpretation of space-time curvature that in turn implies an embedding space, which completely justifies the introduction of the hyper-dimensional hypothesis. Bergmann's caution regarding General Relativity is all the more noteworthy since many scientists had been forced to accept it by the empirical evidence for curvature without a convincing and complete physical interpretation of the concept, a fact which tends to focus attention on General Relativity's most radical aspect, which is, of course, curvature. Another physicist, Robertson, clearly expressed the same notion.

> **Thus Einstein's General Theory of Relativity, which offers an extended kinematics which includes in its geometrical structure the universal force of gravitation, was long considered by some contemporaries to be a <u>tour de force</u>, at best amusing but in practice useless. And now, in extending this theory to the outer bounds of the observed universe, the kind of geometry suggested by the present marginal data seems so repugnant that they would follow Poincaré in postulating some ad hoc force, be it a double standard of time or a secular change in the velocity of light or Planck's constant, rather than accept it.[54]**

The main difference between the biases directed against General Relativity and Kaluza's theory, aside from being one of severity, is that there could be no choice in accepting the validity of General Relativity, whereas, the Kaluza theory rests its case for validation solely on its duplication of the Einstein-Maxwell equations. The experimental evidence clearly supports General Relativity in spite of the radical nature of the concept of curvature while there is not yet any overwhelming corroborative evidence to force the acceptance of either Kaluza's theory or any its various extensions.

### 3.3.3 Defining the Concept of Dimensions

The negative attitude toward the hyper-dimensional theories can be traced still deeper to the Newtonian and pre-Newtonian 3+1 concept of space and time, rather than any four-dimensional concept of space-time, which is basically Einsteinian in its essence. Čapek stated this very fact quite clearly and precisely.

> **.... the homogeneous, passive, rigid receptacle of Euclid and Newton .... is always implicitly or semiconsciously present in our mind when we use the term "space,' and a considerable and constantly renewed effort is necessary to overcome this centuries old habit. Even the mathematical mastery of this theory of relativity does not necessarily guarantee that the habit will be overcome. The widespread misinterpretation of the union of space and time by many physicists and philosophers shows this very clearly.[55]**

As true as this statement is, no reason for a large part of the negative attitude toward General Relativity and, much more emphatically, toward the five-dimensional theories, has ever been found unless a bias toward any theory of space-time that is not four-dimensional is assumed.

The overriding prejudice in our minds of this misinterpretation of the Einsteinian concept of space-time necessitates an investigation into the basic notion of a 3+1 space with time, which is the presently accepted construction of our sensed world. The 3+1 space with time configuration has been taken *a priori* in almost all physical theories, but until the advent of Non-Euclidean geometries, it was probably not assumed with any regard to its 'how?' or 'why?' the configuration was necessary. Many different definitions of dimension have emerged over the years, ranging from Mach's use of the human body,[56] to Menger's formulation[57] and finally to Wheeler's formulation,[58] only to mention a few. The main concern here is with the question of why there are three dimensions, or rather, why there are *only* three dimensions. Arguments vary as to why there are three dimensions, most of which date back nearly a century. Freeman,[59] in his translation of Büchel, offers a good summary of the main arguments in the following table:

| Phenomena | Causes thus Excluded | Remarks |
|---|---|---|
| Bio-topology | n = 3 | Possible only for circular orbit |
| Stability of planetary orbits | n = 4<br>n 4<br>n 3 | Excluded if the potential is to vanish at infinity |
| Stability of atoms | n 3<br>n = 4<br>n 5 | |
| Signal propagation | n even | Non-reverberation condition n=1,3 only remaining possibilities |

It can be accurately stated that there has never been any disagreement with the *a priori* assumption of a three-dimensional space even if it has never been deduced nor implied from any theory. And in this case, all empirical data supports the assumption even if it has seldom been explicitly stated. At best then, one could only hope for a more modest statement on the dimensionality of space, as Abramenko has given us.

> **At the present stage of human knowledge most practical phenomena can be described as if happening in a three-dimensional space and a one-dimensional time, assuming conventionally they are both continuous, or in a world, the line-element of which is expressed by a differential equation of signature (+++-).[60]**

This 3+1 configuration for space plus time has never been proven unique, in that no proof of why or how our space is three-dimensional has been successfully made. So, such a guarded statement as Abramenko's seems entirely in order.

One of the main arguments for the three-dimensionality of space deals with the stability of planetary orbits. Kant recognized the special connection between the inverse-square law and the three-dimensionality of space. Kant also accepted the *a priori* necessity of space. If it were not so, he argued, we could "only be able to say that, so far as hitherto observed, no space has been found which has more than three dimensions."[61] It can be assumed that Kant acknowledged the difficulty of proving the three-dimensionality of space without having assumed three-dimensionality to begin with. Such arguments as this one, regarding the stability of orbits, assume the validity of Newton's mechanics, and thus use the three-dimensionality of space indirectly to prove the three-dimensionality of space. Gregory, on the other hand, has found that such a formalism would yield the inverse-square law in the Newtonian approximation[62] by applying the EIH method of a 4+1 space-time structure. Furthermore, this same argument can be expanded to n-dimensional spaces, where n is arbitrary. Thus we can see that Kant's and other similar arguments are invalid, both for basing their proofs on the assumption which they were attempting to prove as well as the fact that the inverse-square laws are not unique to a 3+1 structure of space with a separate time.

Most other arguments for three-dimensionality of space use, either directly or indirectly, the assumption of a three-dimensional space to prove their argument. The bio-topological argument, which shows the impossibility of a space of less than three dimensions, has been criticized by Tangherlini,[63] who cites that cells located on a multiply connected manifold would not be affected by Whitrow's reasoning that a "typical multicellular animal with an alimentary canal is a torus, .... No configuration of this type is possible in a two- dimensional space."[64] Furthermore, the argument made by Whitrow, although false, sets a lower limit of three dimensions and thus has no affect on the possibility of higher-dimensional spaces. Since such arguments are empirical, they do not nor cannot disprove the possibility of higher-dimensional spaces beyond the 3+1 arrangement. Quite simply, they are unable to prove the intrinsic necessity of their assumed space with time configuration.

In the decades since relativity was introduced, we have come to regard the four-dimensional structure of space-time as accidental with only a very few exceptions.[65] Wheeler's method for determining dimensionality is merely to add another dimension every time we find that we are unable to fit all distances between given points together with the dimensional arrangement that we are using. A surprising result of this method is that we need *only* four dimensions, and no more, to explain the empirical facts of our space-time structure.[66] Even with General Relativity, there is no reference to the necessary dimensionality within the mathematical formalism.[67] This brings us to an important question: If it cannot be uniquely and explicitly proven, either empirically or mathematically, that space is necessarily three-dimensional with time as the fourth dimension, is there a mathematical basis for other spaces of higher dimensions? Several cases can be made for higher-dimensional spaces regarding just this question. Graves has proven that in "a general case of spherical symmetry, only six dimensions are needed" for description if the manifold is already flat.[68] Good has speculated that space-time can have more than four dimensions and uses seven dimensions in his own example.[69] Schlaefli's theorem guarantees that we can always embed some Euclidean space in a flat space of n(n+1)/2 or fewer dimensions.[70] DeSitter's universe, called a hyperbolic space-time, consists of a four-dimensional hypersphere embedded in a five-dimensional manifold (or a six-dimensional flat manifold).[71] Kasner has shown the impossibility of metric fields immersed in flat five-dimensional manifolds and further proven that the solar gravitational field can be represented by a flat six-dimensional manifold.[72] And finally, Rosen and Goldman have discussed the case of a flat five-dimensional embedding space in which a the most "rigid" or restricted behavior of the universe is manifested, since five is "the smallest number of dimensions into which a homogeneous isotropic universe can be embedded."[73]

Mathematically then, it seems easier to show the plausibility of higher-dimensional space-times than it is to prove mathematically or empirically why we must always assume either a 3+1 space with time or a four-dimensional space-time. This does not, however, constitute a proof of the existence of hyper-dimensional space-times. The empirical data as well as our own senses show us, to the best of our present knowledge, that our world is a three space with an extra dimension of time. In like manner, the empirical data and our senses do not absolutely rule out the possibility of hyper-dimensional space-times, nor necessitate a four-dimensional space-time if it can be shown that the empirical evidence merely offers an interpretation of phenomena explained more adequately by higher-dimensional theories. There is no guarantee that the human mind would either sense or perceive a higher-dimension in the same manner that it senses our normal three dimensions of space and one dimension of time, so our human perceptions could nor should not be considered infallible if either physical evidence or a successful mathematical model suggested the reality of a higher-dimensional component of our world.

### 3.3.4 The Compelling Necessity of Higher Dimensions

Perhaps the compelling necessity for the five-dimensional theories can then be found in a compelling *operational* necessity for these theories if such can be demonstrated. Since there is no mathematical reason to limit our choice of space-time coordinates to only four, there is no logical reason why science cannot use the extra degree (or degrees) of freedom that the hyper-dimensional concept seems to offer science to its advantage. If this tactic is successful, science can then assume that a five-dimensional (or higher) embedding space could form the *physical* basis of our world. In other words, if a five-dimensional (or higher) theory can explain our real world in a unique manner, then the existence of a fifth dimension or a fifth component to the space-time continuum should be accepted or at least given due respect until the time that an alternative theory can replace it. The method of using a greater number of degrees of freedom that may correspond to higher-dimensional spaces is not unknown. The DeSitter Universe makes use of five variables corresponding to a five-dimensional embedding space-time. In other cases, extra variables are commonly used to gain extra degrees of freedom without explicitly relating them to higher-dimensional structures. In this sense, Eddington made use of five variables with the fifth appearing as a "phase coordinate" without regarding an equivalently dimensioned space-time structure."[74]

Snyder and Dirac have both used the extra degree (degrees) of freedom in dealing with quantum theory. In his theory, Snyder[75] used a homogeneous quadratic form of equation with five variables. These five real variables were regarded "as the homogeneous (projective) coordinates of a real four-dimensional space of constant curvature (a DeSitter space)."[76] Dirac used a system of matrices forming a complete set representing sixty-two infinitesimal rotations in a six-dimensional space with the forty-two rotations forming a subset corresponding to the four-dimensional space-time.[77] From the historical perspective, Robert Hermann[78] has pointed out that the Yang-Mills idea, which is a direct generalization of the electromagnetic field, can be traced back to Kaluza's theory. He has further stated that

> **One of the main objections to Kaluza-Klein in the 1920's was the extra, fifth dimension that it involved. After fifty years of science fiction (and a lot more mathematics) we are a good deal more broad minded about such things. (The physicist's call it 'internal symmetry degrees of freedom,' while we mathematicians call it 'principle fiber bundles over $R^4$ with compact Lie group as structure group' and make it seem more familiar).[79]**

Judging from these theories and statements, it seems fair to assume that the mathematical use of extra degrees of freedom is a well-established practice in both the domains of the microcosm and the macrocosm.

The use of these extra degrees of freedom is an acceptable feature of physical theory. It is in this manner that the logical operational necessity for the application of a higher-dimensional space-time geometry begins to evolve or emerge from physics itself. If a five-dimensional (or higher) theory were to explain our world, it could possibly not be considered a legitimate worldview because of science's negative attitude toward such theories. However, when a theory using extra degrees of freedom explains our real world in such a way that it can be said to be unique, excluding all other descriptions, it could not be concluded with any certainty that we do not live in a higher-dimensional world that is represented by the number of degrees of freedom. Those theories, which use extra variables associated with higher-dimensional space-times, are but one justification of higher-dimensional theories.

The compelling operational necessity of these theories seems to be related to their greater use in the domain of the quantum than in the macrocosm. The Kaluza theory is a theory of the macrocosm as is the General Theory of Relativity, but the most successful applications of the Kaluza theory seems to have been in the quantum domain. Recognizing this fact, Bergmann stated that

> **The potentialities of the hyperdimensional unified field theories seem great. By playing with the topology of the main field one can produce a great variety of results: the ratio of electrical to gravitational forces or even isotopic spin (Klein).[80]**

Flint and Williamson have both noted that only the adoption of a five-dimensional continuum has been able to unite the quantum with gravitation and electromagnetism, with any sense, in a geometrical way of thinking.[81] Flint has further speculated that "a wider background than that proposed by the four variables x, y, z, and t would make the (Heisenberg Uncertainty) principle unnecessary."[82] Even Pauli, who was dubious about explaining atomism and electric charge using a continuous field approach,[83] stated that

> **The question whether Kaluza's formalism has any future in physics is thus leading to the more general unsolved main problem of accomplishing a synthesis between the general theory of relativity and quantum mechanics.[84]**

In view of such support, even when heavily qualified, it would seem that the operational necessity of the five-dimensional concept is related to its application to the quantum.[85] We therefore not only have a justification for the hyper-dimensional approach to physics, but scientists seem to have identified the realm of nature where that application would be the most effective.

Many attempts have been made to combine the quantum theory with General Relativity by developing a quantum theory of space, time and gravitation. Quantization of the gravitational field in this manner directly opposes the attempts to explain the quantum as a consequence of the field because it presupposes that a unified theory will take the general form of quantum electrodynamics. There is absolutely no sound scientific reason to reduce the field to a more basic quantum. In fact, the differences between discrete and continuous approaches to explaining both matter and physical reality have a long and tempestuous history. Questions regarding which view of reality is more fundamental, the discrete or continuous, have garnered a great deal of debate over the past two thousand years. The quantum versus relativity (or the field) is only the latest version of this age-old debate. The acceptability of strictly quantum approach is questionable since gravitation and space-time are so closely related that any basic change in gravitational theory, such as that proposed by the quantization of gravitation, could not be made without a "profound modification" of the very concept of space-time itself[86] and no such change has been forthcoming let alone validated. Quantum theorists seem more interested in calculated results than the philosophical and physical implications of their theoretical models while a true physics must rely heavily on all of these aspects together. Kaluza's theory has been criticized for its lack of a pre-theoretic or philosophical basis, yet the fundamental nature of the discrete quantum at the expense of the continuous field has no such philosophical basis and criticisms to this effect have seldom been heard in the halls of modern science.

A more natural way of attacking this problem has been demonstrated by relating the fifth component in Kaluza's theory to the quantum in some manner. The fundamental point of developing a five-dimensional theory is its versatility. "The possibility of embedding allows us to define new concepts which might help characterize our geometric structures in an especially revealing way."[87] If Kaluza's theory is completely ignored and forgotten regardless of its early success in uniting electromagnetism and gravitation, albeit in a somewhat synthetic manner, then this versatility and potentiality will have been totally lost to science. The ultimate progress of science and its reputation for objectivity could well be the losers if attempts to unify the laws of nature proceed only from the quantum perspective.

A large part of the problem in any logical argument against the five-dimensional theories is that the term "space-time curvature" is a misconception which leads scientists to falsely regard hyper-dimensional embedding spaces as something which they are not. Space need not be curved in a higher dimension when all properties of that space are intrinsic, but that does not mean that a higher-dimensional framework is unjustified when it can account for phenomena which cannot be explained by the suspected intrinsic structure of a four-dimensional space-time. However, no manner of justifications can change the deep-rooted psychological bias against spaces of five or more dimensions. Čapek feels that

> **Our subconscious is far more conservative than we are willing to admit, and this is true not only for our emotional subconscious, but of the intellectual one as well. That is why Newtonian-Euclidean habits of thought will - if not always, for a considerable time - appear more natural to mankind than the new modes of thought which require so much effort and vigilant analysis.[88]**

We fool ourselves by allowing our psychological biases against five-dimensional theories to cloud our intellectual considerations of such space or space-time structures. Marie-Antoinette Tonnelat also recognized and noted this problem.

> **Many physicists still consider, in spite of everything, that the success of this sort of formalism is rather artificial. The success of a language adapted to a five-dimensional manifold is, according to them, only a way of concealing the lack of developments truly adaptable to the four-dimensional universe.[89]**

If Čapek is correct, then there is a strong psychological reason for assuming a bias against attempting five-dimensional theories, and those scientists mentioned in Tonnelat's historical and conceptual publications of the subject, who consider five-dimensional theories to be merely a "lack of developments truly adaptable to the four-dimensional universe," may still be so encumbered by the language of Newtonian mechanics as to be unable to even consider the plausibility of Kaluza's theory and those theories that followed. Both Graves[90] and Capek[91] agree to some extent that we must purge our Newtonian conceptions and associations, forgetting Newton and what he has told us about gravitation, before we can truly see General Relativity. This suggestion would be even truer for the extrapolation and extension of General Relativity into a higher-dimensional framework. Perhaps only then could we realize the extent to which General Relativity and Special Relativity represent a revolution in the scientific worldview. And perhaps then we could more readily accept the possibility that radical concepts such as the five-dimensional structure of space-time used by Kaluza may represent the logical conclusion to the revolutionary concepts introduced by Einstein.

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Last Updated 22 December 1999

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