BUBBLE, BUBBLE, TOIL AND TROUBLE
A fresh look at relativity, uncertainty and compatibility

By **James E. Beichler**

Reality is in the eyes, and perhaps the mind, of the beholder. What we see or otherwise sense in the world around us is what we as sentient beings accept as reality. However, our sensual perceptions are modified by our minds. An artist painting a picture portrays his individual sense of reality while a musician composing a symphony describes reality and the harmony of the universe after his own manner. Theologians, philosophers, biologists and other thinking individuals may wish to interpret the physical universe of experience and the reality that it represents in still other terms. But the physicist claims the right to discover the nature of physical reality within the limits of the human mind's ability to perceive the world about us. While other disciplines offer views or opinions of reality from their own perspectives, physics claims to seek a universal perspective, which is as free from bias and opinion as is humanly possible. Yet, in all of these attempts there seems to be a fundamental assumption that there is some basic form of reality beyond the mind's perception of it, and this notion bears investigation.

There are two fundamental views of reality within the scientific community of physicists and they have been at odds with each other since their initial discovery. In the first view, it is held that physical reality consists of a space-time continuum as represented by the theories of relativity. Relativity presupposes that exact measurements can be made on the quantities it describes. In the other view it is held that there is an inherent uncertainty in our measurement of specific quantities in nature and we can never determine the exact nature of our physical world. This second view is based upon the uncertainty principle in its several forms.

Philosophical interpretations of the uncertainty principle range from one extreme whereby the existence of reality is dependent upon some type of conscious perception or observation to a conviction that the uncertainty is due to the act of measurement itself. This second extreme implies that there is a reality beyond human observation and that it might be possible to go beyond the inherent limits set by the uncertainty principle to discover the true nature of reality. In either interpretation, the uncertainty principle places limits on our scientific knowledge of reality and nature is thus rendered indeterministic. This proposition is opposed by relativity theory, which places no such limits on our knowledge of physical reality and thus assumes that nature is inherently deterministic.

The philosophical basis of this schism of belief has evolved primarily from what has become known as the EPR paradox. Einstein, Podolsky and Rosen published a paper in 1935, which challenged the Copenhagen Interpretation of quantum theory and the uncertainty principle. EPR claimed that quantum theory was incomplete and it was at least possible to infer both the position and momentum of a material object at the present level of scientific understanding even if they could not be directly measured. It assumed the underlying reality of the physical world independent of the act of observation. The feasibility of later scientific advancements, which would lead to a better understanding of the problem and true simultaneous measurement of position and momentum, was also implied in the EPR argument.

Bohr and Heisenberg supported the other extreme of interpretation of uncertainty. This is known as the Copenhagen interpretation. These physicists supported a view that the uncertainty principle placed limits upon our knowledge of physical reality since we could never measure position and momentum simultaneously. The problem of exact simultaneous measurement was not in the instruments of measurement and could thus not be overcome by using more refined instruments representing further advances in the theory of the quantum. So quantum theory as represented by the uncertainty principle was complete, the problem of measurement is in nature itself, a property of reality.

Bohr's reaction to Einstein's accusations of incompleteness was immediate, but the debate has continued to a greater or lesser extent over the ensuing decades. To this date, the argument has not been resolved to the satisfaction of all physicists, philosophers and scholars, yet it has not proved to be a hindrance to the physicist. On the contrary, the philosophical debates over this issue have lead to new experiments to test these viewpoints. One important interpretation of EPR that has had a stimulating influence on experimental physics is the proposal that there are hidden variables underlying our present theories. The discovery of these hidden variables could render a more complete description of nature than could be afforded by the uncertainty principle alone. A great deal of literature, representing both philosophical and physical researches, has been accumulated upon the foundations of such hidden variable interpretations of the EPR paradox.

The wealth of literature on this approach has been highlighted by the work of von Neumann and Bell. In 1932, John von Neumann offered a mathematical proof that hidden variables are impossible within the framework of quantum theory. In 1952, David Bohm questioned the relevance of von Neumann's proof, reopening the debate, and in 1964 J.S. Bell argued that von Neumann's proof and similar analyses left the "real questions" presented by the hidden variable approach untouched.

Bell's theorem drew a distinction between the local and non-local features of uncertainty. Hidden variables, which only affected nature in the immediate vicinity of an event, could produce observable results that contradicted the predictions of quantum mechanics. However, the substratum of space and time does not necessarily depend upon such local hidden variables. Instead, other hidden variables, which acted non-locally and thus affect events over a much greater range, could be the contributing factors of our deterministic world. When such non-local parameters vary, they affect the overall substructure of the world. Bell's theorem suggests that experiments to distinguish between local and non-local properties governed by quantum mechanics are feasible.

Relativity theory implies that there are no 'localizable' particles of matter that conform to Bell's model without affecting the substructure of our world. However, recent experiments designed to test for 'Bell's inequalities' seem to support the Copenhagen interpretation in contradiction to relativity theory and the possibility of hidden variables that conform to Bell's theorem. Different entities within quantum mechanical systems seem to communicate across space and time, and thus act non-locally, defying the model of space-time which represents relativity theory. It would seem that Bell's argument suffers by its strict limitation of EPR to a hidden variable interpretation.

In Bell's own words, EPR implied the existence of hidden variables and other interpretations of EPR were not taken into account.

> The paradox of Einstein, Podolsky and Rosen was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables. These additional variables were to restore to the theory causality and locality. In this note that idea will be formulated mathematically and shown to be incompatible with the statistical predictions of quantum mechanics. (Bell [1964] 1987, 14)

In this statement the word 'incompatible' stands out for its lack of definition while its use implies that uncertainty and relativity should be compatible for a 'complete' description of nature. Compatibility among the various laws of physics must then be considered a desirable quality.

For example, the uncertainty principle is compatible with Newtonian mechanics since the results of quantum mechanics approach Newtonian mechanics in the proper domain of application. This property of the theory was named complementarity by Bohr. Special relativity is also compatible with Newtonian results when speeds are slow compared to the speed of light and general relativity does not differ appreciably from Newtonian gravitation when dealing with all but the strongest gravitational fields. With all of these well-documented examples of compatibility, it would seem that there would be some measure of compatibility between relativity theory and the uncertainty principle even though they seem to prescribe different notions of reality. There are relative forms of quantum theory that accurately describe physical phenomena and there have been numerous results combining special relativity and quantum theory, but fundamental differences between these two areas of physics still remain even though these successes suggest the compatibility of relativity and uncertainty.

The extent to which relativity and the quantum theory are compatible has never been determined and the conditions of compatibility have never been established. A common philosophical meeting ground between uncertainty and relativity will not be found unless some criteria of compatibility are established. At the most basic level, it would be necessary for any criteria to take into account the fact that different theories must yield the same physical results in areas of common applicability. Theories that explain physical reality in different realms of nature must give the same results in areas where the domains of their explanation overlap. The abstraction of this notion leads to the possibility that different theories are compatible if the mathematical formulas by which those theories model nature can be derived from each other. Such derivations not only represent a criterion for compatibility but also offer a measure of the degree to which different theories are compatible by establishing the conditions of compatibility.

Special relativity and quantum mechanics share a common realm in the microscopic world of fundamental particles. Therefore, it would seem that the basic formulas of relativity should be derivable from the uncertainty relationships and uncertainty should be derivable from the properties of relative space and time. Yet this task has not been accomplished and probably never attempted.

Clues to the method of deriving relativistic formulas from the uncertainty relationships are suggested by a close examination of the physical quantities found in the various statements of the uncertainty principle. The better-known relationship

$$\Delta x \, \Delta p \geq h$$

exhibits the variables position in space and momentum which can be defined as material change of position in space. The second relationship

$$\Delta E \, \Delta t \geq h$$

refers to position in time and energy. When the quantities in these two relationships are compared to one another, one may suspect that energy is related to time in a manner similar to momentum's relation to space. The mathematical relationship is trivial, but the philosophical relationship is fundamental. If momentum represents material change of position in space, then energy represents material change of position in time. These interpretations of momentum and energy are not those that are usually expressed, but are consistent with the uncertainty relationships in which they are found. There is also a precedent for this view of energy in the field of thermodynamics where the dissipation of energy is considered "time's arrow."

When viewed in this manner, the uncertainty relationships seem to make separate cases for time and space. On the other hand, the fundamental structure of physical reality is a combined space-time according to relativity. This fact alone would place uncertainty at odds with relativity, but it also suggests the proper procedure to bring the theories together. Recombining space and time as expressed in the uncertainty relationships should lead to relativity.

Since the combination of factors, whether '$\Delta x \, \Delta p$' or '$\Delta E \, \Delta t$', are each equal to or greater than the same quantity, h, they can be set equal to each other.

$$\Delta x \, \Delta p \geq h \leq \Delta E \, \Delta t$$

$$\Delta x \, \Delta p = \Delta E \, \Delta t \, .$$

And, by rearranging terms,

$$(\Delta x / \Delta t) \, \Delta p = \Delta E \, .$$

At this point it is the ratio $\Delta x/\Delta t$, which bears closer analysis. The ratio looks something like a speed, but the fact that it represents instead a ratio of two independent uncertainties does not allow such a hasty interpretation. However, since we are dealing with uncertainties that have limits, there is no logical reason why a limit to the ratio cannot be established.

There are three possibilities for such a limit. The ratio $\Delta x/\Delta t$ can either be less than c, equal to c or greater than c. It is convenient then to introduce a new concept called relative uncertainty, by which the complementary quantities of quantum mechanics vary in such a way that the ratio of Δx to Δt would have a maximum value of the speed of light, or $\Delta x/\Delta t \leq c$. The alternative case where $\Delta x/\Delta t > c$ can be called absolute uncertainty. There is no why reason these ratios cannot be so named while the justification of such evaluations will soon become apparent.

The application of relative uncertainty leads to the desired results, in this case, the derivation of the Lorentz-Fitzgerald contraction. After applying the condition of relative uncertainty to the combined uncertainty relationships, we have

$$c\ \Delta p = \Delta E$$

Since the condition of relative uncertainty has consumed the only factor of Δx and the contraction involves a distance, we must introduce a new factor of Δx to each side of the equation to "renegotiate" the uncertainty.

$$c\ \Delta p\ \Delta x = \Delta E\ \Delta x$$

Letting Δp go to $m_o c$ will later allow us to vary ΔE on the right hand side of the relationship and Δx on the left. If a quantity on one side is certain, then the uncertainty is inherent in the complementary quantity.

$$c\ \Delta(m_o c)\ \Delta x = \Delta E\ \Delta x$$

There is no uncertainty in either m_o or the speed c, so these can both removed as uncertainties, leaving

$$(\Delta x)\ m_o c^2 = \Delta E\ \Delta x\ .$$

On the other hand, the energy of the system can be represented by the classical expression for the kinetic energy of an object since the two sides of the relationship must be treated differently.

$$(\Delta x)\ m_o c^2 = (1/\,2)[\Delta(m_o v^2)]\ \Delta x$$

In the relative case we are not measuring the position of a single point particle as is normally done in the quantum mechanical case. Instead, we are measuring the relative positions of the endpoints of an extended object, i.e., the length of an object. Where L is the length at the speed v, so Δx becomes L on the right hand side of the equation.

In a similar manner, the quantity of Δx on the left hand side of the equation becomes $(L_o\text{-}L)$, giving a positive value for the length, which is physically necessary.

$$(\Delta x)\ m_o c^2 = (1/\,2)\ [\Delta\ (m_o v^2)]\ L$$

Since m_o is not uncertain, it can be withdrawn from the uncertainty term once again.

$$(\Delta x)\ m_o c^2 = (1/2)\ m_o\ \Delta(v^2)\ L$$

If the original length of a "test" object is L_o, then the length at velocity v would be L. L_o would become L + ΔL, or rather the uncertainty in the new length at v would be L_o-L, the amount contracted. Then, by algebraic manipulation

$$(L_o - L)\ m_o c^2 = (1/2)\ m_o\ \Delta(v^2)\ L$$

$$[(L_o - L)/L]\ m_o c^2 = (1/2)\ m_o\ \Delta(v^2)\ .$$

And,

$$[(L_o/L) - 1]\ m_o c^2 = (1/2)\ m_o\ \Delta(v^2)$$

$$[L_o/L) - 1] = (1/2)\ [m_o\ \Delta(v^2)/(m_o c^2)]$$

$$(L_o/L) = 1 + [(1/2)\ m_o\ \Delta(v^2)/(m_o c^2)]$$

$$(L_o/L) = 1 + [(1/2c^2)\ \Delta(v^2)]$$

$$(L_o/L) = 1 + \Delta v^2/2c^2\ .$$

The quantity on the right of this relationship represents the first two terms of a power series expansion. The remaining terms of the expansion contribute to the uncertainty in the speed. Substituting the unexpanded expression yields

$$(L_o/L) = [1-(v^2/c^2)]^{-1/2}\ .$$

Finally, by rearranging the variables, we have

$$L = L_o\ [1-(v^2/c^2)]^{1/2}\ ,$$

which is the correct formula for the Lorentz-Fitzgerald contraction. This derivation is admittedly ad hoc and non-rigorous from a mathematical point of view, but still suggests the degree of compatibility between special relativity and uncertainty. The other formulas, which are usually associated with special relativity, can be derived in similar manners. In each instance relative uncertainty must be involved. It would then seem that the condition for compatibility of uncertainty and relativity is the fact that the ratio Δx/Δt is less than or equal to c.

The inverse operation of deriving the uncertainty principle from space-time considerations is no more difficult, but none-the-less different. It also starts from the same premise of relative uncertainty.

$$\Delta x/\Delta t \leq c$$

$$\Delta x \leq c\ \Delta t\ .$$

From classical physics we have the relationship

$$(dx)^2 = c^2(dt)^2 ,$$

but relativistic considerations whereby c is a limiting speed will allow this to change to

$$(dx)^2 \leq c^2(dt)^2 .$$

In any relativistic case, it can be expected that a material particle or photon has a well defined position in space-time. So, if a material particle or photon is moving through a distance dx during the time interval dt and there is an uncertainty in one of these quantities, there will be a corresponding uncertainty in the other quantity. The previous relationship can then be rewritten as

$$(\Delta x)^2 \leq c^2(\Delta t)^2 .$$

This relationship is mathematically equivalent to the above statement representing relative uncertainty. But from relativity theory we also know that $E = mc^2$ and thus $c^2 = E/m$

$$(\Delta x)^2 \leq (E/ m) (\Delta t)^2 .$$

We also have the relationship describing the deBroglie wavelength of a particle, which equals h/p. If we allow the possibility that the uncertainty in x is equal to the deBroglie wavelength, this last relationship can be written as

$$(h^2/ p^2) \leq (E/ m) (\Delta t)^2 ,$$

and then

$$h^2 \leq (\Delta p^2/ m) (E) (\Delta t)^2 .$$

But this reduces to

$$h^2 \leq (\Delta E) (E) (\Delta t)^2$$

since $E = p^2/m$ in the relativistic case. Allowing for an uncertainty in energy, we now have

$$h^2 \leq (\Delta E)^2 (\Delta t)^2$$

and therefore

$$h \leq \Delta E \, \Delta t ,$$

which is the correct relationship for uncertainty. Like the case for Lorentz-Fitzgerald contraction, there are inherent deficiencies in this derivation. However, the fact that the derivation can be completed at all is evidence that uncertainty can be derived from relativistic principles under the proper conditions. Therefore, there is some degree of compatibility between special relativity and uncertainty.

In the past, criticisms were leveled at Einstein for supporting the validity of the experimental evidence of quantum mechanics while criticizing the Copenhagen Interpretation. The argument for compatibility supports Einstein's correctness in doing so. Einstein may have had an intuitive feel for what has been called relative uncertainty without ever having articulated the concept. There is an underlying assumption in the EPR argument that the interactions described were occurring in a relative space-time. It was also implied in EPR, just as it is assumed in relativity theory, that any two particles could be used to establish a relative space-time framework. In other words, if the universe were limited to only two material particles, a relative space-time would still exist. A precedent for this view was established in Mach's principle and there is no reason to believe that Einstein would deny this view.

This view can be extrapolated to the statement that any two particles in our own universe are all that is needed to establish a relative space-time. When Einstein argued in EPR that the reality of two particles was still assured after their mutual interaction, even though the uncertainty principle would not allow the simultaneous measurement of both their positions and momentums, he was merely supporting the view that both of these particles had definable positions and momentums because in principle they still established a relative space-time framework. This assumption, the theoretical basis of relativity theory, was the fundamental unstated basis of Einstein's refusal to accept the Copenhagen Interpretation of quantum mechanics. To accept the Copenhagen Interpretation would have amounted to a denial of the theoretical basis of relativity itself. Quantum mechanics was incomplete and would remain so in Einstein's eyes until science could explain how the complementary quantities of the uncertainty principle could remain uncertain and still allow the existence of a definable space-time framework, independent of an observer.

One early way to avoid these problems was to dissociate Einstein from the EPR paper. In this regard, there have been some criticisms that the EPR paper was not written by Einstein or that he had varying amounts of influence over its final form, but it cannot be argued that he disagreed with its basic propositions. The points of debate put forward in the paper are consistent with arguments that Einstein made against the popular interpretation of the quantum theory both before and after the publication of EPR. Einstein clarified his own arguments in later publications on the paradox.

The possibility of hidden variables suggested another solution to this dilemma. They could guarantee the existence of the necessary space-time framework even though position and momentum could not be simultaneously defined or measured. This line of reasoning has proven fruitful for experimental physics, but has not yet proven either the existence or non-existence of hidden variables. Should such variables exist, they must be so far beyond our present level of comprehension that science has no idea how they would appear or what form they would take.

Compatibility offers yet another avenue of interpretation which negotiates the differences between the seemingly incompatible points of view of Einstein and Bohr. In one sense, quantum mechanics is incomplete because there is a condition, defined as absolute uncertainty, which does not allow a strictly definable space-time framework. This view accounts for the most extreme version of the Copenhagen Interpretation whereby reality is dependent upon the act of observation itself. Simultaneously, quantum mechanics is complete with respect to another condition as expressed by relative uncertainty. When the ratio of Δx to Δt is either less than or equal to c, quantum mechanics and special relativity are compatible and the relative space-time framework is completely defined given any two or more material particles.

The existence of compatibility also begs for a reinterpretation of the quantities involved in the uncertainty relationships. Since the uncertainty formulas separate space from time, $\Delta x\ \Delta t \geq h$ can be considered a purely spatial relationship while $\Delta E\ \Delta t \geq h$ is purely temporal. Δx represents an uncertainty of position in space and Δp represents the corresponding uncertainty of a change in that position. Classically and relativistically, a change in position must include a representation of both the matter undergoing change in position as well as the duration over which that change occurs. Matter is clearly represented in momentum, but time is only indirectly represented in the portion of momentum that represents the rate of change of position.

If momentum is to be defined as an independent and elementary quantity when time has been deleted from consideration, the quantity Δp must represent the uncertainty in material change of position independent of time. The total quantity $\Delta x\ \Delta p$ must then represent the total uncertainty found in any attempt to determine a fixed position while position is changing without reference to changing time. This concept is difficult to understand since it would seem that any change in position could only occur over some time interval, however small. This uncertainty relationship could alternatively be considered the limit to which position in space can be determined without reference to time.

The second relationship considers the uncertainties of energy and time. This relationship represents an attempt to find position in time independent of position in space. While Δt represents the uncertainty of position in time, ΔE represents the corresponding uncertainty or the extent to which time can be measured without regard to spatial considerations. Energy should then be considered a model of material change of position in time independent of spatial considerations, just as momentum represents a material change in spatial position.

In essence, these two relationships are measures of the extent to which space and time can be measured independently. They are almost an attempt to return to the Newtonian concept of pure space and pure time in a modern world of science that has already proven the attempt futile. When time is independent of space, energy is space-like or acts as a substitute for space. When space is measured independent of time, momentum assumes the role of a time-like quantity. Only when we combine the two relationships can we recreate space-time and again have the formulas of special relativity. Since the uncertainty relationships are a model of the extent to which space and time can act independently, Planck's constant becomes the dissociation constant for space-time.

In this interpretation of physical reality, the ratio $\Delta x/\Delta t$ holds a special position that bears further analysis. When so analyzed, the ratio can be interpreted in several ways. The statement $\Delta x/\Delta t \leq c$, which has been designated relative uncertainty, does not necessarily indicate that a particle or wave is traveling at or below the speed of light. The quantity c is a universal constant, just as it is in the relationship $E = mc^2$. In this later formula, the use of c as a constant does not mean that matter must travel at a speed of c^2 to be converted to energy. It is sufficient that a particle moves at or below the speed of light to yield relative uncertainty, but it is by no means necessary. There may be other physical conditions or hidden variables that yield relative uncertainty regardless of the rate of change of position of any given particle.

On the other hand, rates of motion greater than that of light are sufficient to yield $\Delta x/\Delta t > c$, but are not necessary for absolute uncertainty. There may be either physical conditions or hidden variables that lead to a situation of absolute uncertainty without regard to the concept of speed as the rate of change of position. There are neither philosophical nor experimental reasons to deny the possibilities of these other physical interpretations of $\Delta x/\Delta t$ at this time. So there still remains a possibility that hidden variables play an important role in quantum theory, even though hidden variables are no longer necessary for the compatibility of special relativity and quantum theory.

There is definitely room for further physical analysis and speculation based upon compatibility. Perhaps hidden variables which act locally, contributing to the substructure of space-time, correspond to the case whereby $\Delta x/\Delta t > c$ when c does not correspond to a rate of change of position. Hidden variables which act non-locally, while contributing to the substructure of space-time, may well correspond to the case in which $\Delta x/\Delta t \leq c$ and c is not a measure of the rate of change of position. And perhaps the results of recent experiments which support neither 'Bell's inequalities' nor Einstein's interpretation of quantum mechanics and special relativity correspond to the case where $\Delta x/\Delta t > c$ with c acting as a true rate of change of position.

By differentiating between these possibilities while demonstrating how they relate to the present laws of nature, at least a greater understanding of reality is possible. And only a greater understanding of the nature of reality can lead to observational and experimental results that can decide the true reality of nature.

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BIBLIOGRAPHY

Bell, J.S., "On the Einstein-Podolsky-Rosen paradox," Physics, 1 (1964): 195-200; reprinted in Bell, Speakable and Unspeakable in Quantum Mechanics (Cambridge: Cambridge University Press, 1987): 14-21.

Bell, J.S., "On the problem of hidden variables in quantum mechanics," Reviews of Modern Physics, 38 (1966): 447-452; reprinted in Bell, Speakable and Unspeakable: 1-13.

Bohm, David, Physical Review, 85 (1982): 166, 180.

Bohr, Niels, "Can the Quantum-Mechanical Description of Physical Reality Be Considered Complete?" Physical Review, 48 (1935): 696-702.

Deltete, Robert and Reed Guy, "Einstein and EPR," Philosophy of Science, 58 (1991): 377-397.

Einstein, Albert, B. Podolsky and N. Rosen, "Can the Quantum-Mechanical Description of Physical Reality Be Considered Complete?" Physical Review, 47 (1935): 777-780.

Fine, A., The Shaky Game: Einstein, Realism and the Quantum Theory (Chicago: University of Chicago Press, 1986).

von Neumann, J., Mathematische Grundlagen der Quanten-Mechanik (Berlin: Julius Springer-Verlag, 1932); In English translation (Princeton: Princeton University Press, 1955).

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Top of Page
YGGDRASIL Homepage

Go to Issue 2