The Five-Dimensional Continuum Approach to a Unified Field Theory

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This thesis was completed in 1979. It now appears in a revised form, although the content itself has not been changed. When it was first written, the days of the hyper-dimensional unified field theories had long past and no one in the scientific community seemed interested in such ideas. No work had been conducted in this area of research since the mid-1950s. However, the idea was still "in the air." Unknown to the author, other scientists were also beginning to take notice of Kaluza's work. The most notable of these scientists was Abdus Salaam who, with a small group of scientists, was again considering the possibility of a five-dimensional universe as described by Kaluza in 1921.

During the early 1980s,the Kaluza-Klein theory was revived to explain the presence of gauge symmetries in the Grand Unification Theories (GUTs). These symmetries were associated with the extra dimensions in the physical model, eleven in all, which were required by the GUTs. Supergravity fields also utilized a Kaluza-Klein formulation of eleven dimensions, but this time for the purpose of the unification of matter in the form of the Yang-Mills field and the Einstein-Maxwell field. The GUTs attempted instead to unify the electroweak force described by the Salam-Weinberg model and the strong nuclear force. Both theories faced serious difficulties, but were popular among scientists. Each utilized an eleven-dimensional Kaluza-Klein model, so it was hoped that the two theories would eventually merge into a single complete model of all the natural forces.

In the meantime, a third area of theoretical research was developing which would steal some of the thunder from these attempts at unification. These attempts were called string theories. It was discovered that the string theories could be unified by a supersymmetry that could be represented in either a ten or twenty-six dimensional Kaluza-Klein framework, and thus the superstring theories evolved. The purpose of the superstring theories is nothing less than the total unification of all the natural forces as well as the quantum and relativity and in this quest the 'cylindrical condition' of the original Kaluza theory, as later modified by Klein, was the perfect vehicle for describing the sub-atomic strings.

Given these new advances, the Kaluza-Klein theory had again become popular enough for other authors to come forward by the 1990s and offer their own views of the genesis of the hyper-dimensional theories. In his book *Hyperspace*, Michio Kaku renders the Kaluza theory into its own niche in the history of science, while the book *Modern Kaluza-Klein Theories*, a collection of reprinted essays and articles which have had a direct influence on the development of superstring theories, includes copies of both Kaluza's and Klein's original papers from the1920s. The editors of this book have also included a historical essay on the development of the five-dimensional hypothesis and its relation to the superstring theories. However, these authors let the end of the story, the development of superstring theories, guide and influence their histories of the five-dimensional hypothesis. So their histories are gravely incomplete and thus inadequate for truly historical purposes. Their histories are Whiggish since they have ignored the many other advances in hyper-dimensional work that do not seem to pertain directly to the theoretical structure of superstrings at first glance. They only tell of the work that fits their pre-conceived notion of what higher-dimensioned manifolds could be. Superstring theorists have adopted the Kaluza-Klein theory without regard for, and probably without any knowledge of, the criticisms that the theory originally faced. At least there is no evidence that the superstring theorists have considered other possible five-dimensional theories. Some scholars

may deem such a practice adequate for scientific purposes and references, but it is woefully incomplete for historians and philosophers. However, scientists could learn a great deal from a more thorough and comprehensive study of the development of hyper-dimensional theories as a whole.

The brief history that Kaku proposes, dating all the way back to the original work of Bernhard Riemann and William K. Clifford, is filled with errors that any knowledgeable historian could not allow to pass without comment. The fact that Kaku is a physicist attempting to write history is not an excuse for making gross historical errors. This thesis represents the only modern history of five-dimensional theories taken as a whole. It is not aimed at proving or demonstrating the historical necessity of later developments in science or any particular point of view. It is precisely for this reason that the thesis has not been updated to include developments in physics since 1980 during its revision. Present historians or scientists have added nothing new or even constructive to the historical view, so no changes have been made to the content of the thesis. Nothing new need be added to the thesis to make it more complete. Given the lack of accurate information and misinformation that is presently available to readers, it seems that much of great value seems to have been lost and or forgotten, so the publication of this thesis is necessary. Hopefully, reading this thesis will enlighten those who have erred in their own studies, generate new ideas about the possibility of higher-dimensional manifolds and spur the interest of others not yet involved in this interesting and significant area of physics.

CHAPTER I

THE HISTORICAL DEVELOPMENT

1.1 KALUZA AS AN ORIGINATOR

Even though Albert Einstein's name is usually associated with the search for a unified field theory, the first attempts to develop such a field theory were made by Herman Weyl in1918 and Theodor Kaluza in1921. In fact, Einstein did not at first believe that developing a unified field theory was either necessary or expedient. So the notion of identifying and modeling a single physical field from which the known physical forces could be derived did not originate with Einstein. Both Weyl's and Kaluza's attempts in this endeavor have fathered the main groupings of unified field theories over the years. Weyl sought to alter the geometry of the continuum while maintaining the number of dimensions at four. He noted that Riemann's geometry went only "halfway towards attaining the ideal of a pure infinitesimal geometry,"[1] so he introduced a gauge system into the space-time geometry as a remedy to that oversight. In his new geometry, the parallel transfer of a length in the field would allow a change in the basic unit of length according to the gauge. This change accounted for the presence of distant-curvature and thus allowed the introduction of electromagnetism into the metric of space-time curvature. Unfortunately, Weyl's initial attempt was found to yield physical consequences that were contrary to experimental evidence and thus proved a failure. Yet his work with affinely connected spaces, spaces governed by the parallel displacement of a vector, was later extended by Eddington, Einstein and others. In 1944, Erwin Schrödinger used the affine connection as the basis for his own unified field theory. Einstein arrived at virtually the same theory following a different line of reasoning, his non-symmetric approach. The theory that evolved from these efforts, the Einstein-Schrödinger Non-

Symmetric Field Theory, has come to represent what is presently regarded as the most advanced 'classical' unified field theory.

On the other hand, Kaluza's theory retained the purely Riemannian space-time of General Relativity while extending the field structure by the addition of an extra dimension. Like Weyl's basic concepts, Kaluza's ideas have led to many modifications and extensions, but unlike Weyl's theory the five-dimensional structure built by Kaluza has never been proven wrong and still stands as an independent theory as it was originally conceived. It has yet to be found completely unsound, even though it has run into serious criticisms that have hampered its credibility and acceptance within the scientific community.

The original version of the theory was presented in an article entitled "Zum Unitätsproblem der Physik."[2] The complete theory comprised only seven pages, but this was enough to capture the imaginations of Einstein, Oskar Klein and others. This one article seems to be the major, as well as the only article published by Kaluza on the five-dimensional theory. All references made to Kaluza by other scientists and authors note only this paper. Yet there is evidence that Kaluza was working on the five-dimensional theory at least two years before the article was published and perhaps for a time afterwards. In a letter dated 21 April 1919, Einstein made the following remark regarding Kaluza's earlier communication of the five-dimensional theory to him

> **... der Gedanke, dies (elektrischen Feldgrossen) durche eine Funfdimensionale Zylinderweld zu erzeilen, ist mir nie gekommen und durfte uberhaupt neu sein. Ihr Gedanke gefallt mir zunachst ausserordentlich.**[3]

Einstein thus "encouraged [Kaluza] to pursue such an approach, submitting that this was an entirely original point of view."[4] Einstein realized that Kaluza was above his position as a 'privatdozent' at Königsberg, based upon his correspondence with Kaluza regarding the five-dimensional theory, and later recommended him for a better position. These statements and actions may seem a contradiction considering comments that Einstein included in his book *The Meaning of Relativity*, first published in 1922. By1922, Einstein came to recognize and accept the unsatisfactory inclusion of the electromagnetic field in the equations of General Relativity as the basis of seeking a unified field theory, but he stated that

> **A theory in which the gravitational theory and the electromagnetic field do not enter as logically distinct structures would be much preferable. Hermann Weyl and Theodor Kaluza, have put forward ingenious ideas along this direction; but concerning them, I am convinced that they do not bring us [any] nearer to the true solution of the problem.**[5]

At first, Einstein made no active attempt to develop a unified field theory based on the affine connection that Weyl had founded and Eddington had since modified. Einstein's first written papers on the five-dimensional field theory didn't appear until 1927[6] and these added nothing new to the theory beyond Klein's development[7] of the year before. So, despite his early encouragement and private recognition of Kaluza's ideas, Einstein was never totally convinced of the efficacy of the five-dimensional approach to unified field theories even during his correspondence with Kaluza. Einstein later added his own theoretical twists to Kaluza's theory, but eventually discarded the five-dimensional approach to a unified field theory.

Kaluza seems to be the originator of the five-dimensional concept in that he developed the first true five-dimensional theory. His theory, with Klein's modification and extension, forms the basis of many geometries of this type. Keeping this fact in mind, several historical assumptions can be made regarding Kaluza's contribution to science and his place in history. Nearly all of the later papers dealing with five-dimensional and higher geometries refer to Kaluza's original paper, if they make any references at all. Only DeBroglie, in his paper of 1927, "L'Univers A Cinq Dimensions et La Mécanique Ondulatoire,"[8] attempts to give credit to another scientist in the person of H.A. Kramers. Subsequently, Klein,[9] in a letter to *Le Journal de Physique et le Radium*, pointed out that DeBroglie was mistaken and that Kramer's paper, as cited, dealt only with the quantum theory and had made no mention of the five-dimensional space-time. DeBroglie quickly admitted his error.[10] There has since evolved a general consensus among scientists and scholars that Kaluza originated the five-dimensional concept.

Kaluza also seems to have worked alone, seems never to have published any other papers on his theory and, for all intents and purposes, seems to have ended his theoretical work on the five-dimensional concept with the publication of his paper. The only suggestion that Kaluza later collaborated with others on his theory or continued work on the theory alone can be found in a statement by Wolfgang Pauli in his book *The Theory of Relativity*. Pauli stated, "Kaluza and Klein derived, however, a further interesting result. They computed the scalar P of the curvature tensor, which corresponds to the particular five-dimensional metric ... and found that $P=R+(1/4)F_{ik}F^{ik}$ ... ."[12] This equation, at least in this form, does not appear in Kaluza's paper, which would lend some support to the possibility of Kaluza's later development of the theory and his direct collaboration with Klein and others. Yet, there is neither evidence nor even a hint that Kaluza and Klein worked together. Pauli's statement is only circumstantial evidence at best. Their possible collaboration cannot be ascertained without an extensive search, which would include Kaluza's personal papers and records. Klein, of course, did extend Kaluza's theory and it would be hard to imagine that he wasn't in contact with Kaluza at some point to ask for Kaluza's help and opinion on the further development of the theory.

1.2 PRE-KALUZAN FIVE-DIMENSIONALITY

Kaluza was, and still is, considered to be the originator of the concept of five-dimensional unified field theories. This is true since he began a historical tradition of such theories as were developed as further generalizations of General Relativity. However, it would be false to assume that he alone originated the idea of a five-dimensional space-time. In fact, it is possible that he may have had knowledge of at least one previous five-dimensional theory, which was, however, not an extension of General Relativity. It is further possible that the five-dimensional concept was not completely unknown and probably suspected by others before Kaluza's work became public knowledge. The notion of four-dimensional spaces independent of the single dimension of time was nearly a half-century old and quite popular just prior to the development of General Relativity. So the possibility of higher-dimensional space-times may well have been lurking in the thoughts of other scientists to a greater degree than would be indicated by the lack of early development of the idea. Twenty-five years after Kaluza's original publication, H.C. Corben[13] attempted to unify gravitation with the electromagnetic field by five-dimensionalizing Special Relativity. In this attempt, Corben cited previous work by Kaluza and Klein as well as Gunnar Nordström of Helsingfors. By so doing, Corben became the only scientist found to have given verifiable credit for the five-dimensional concept to someone other than Kaluza (or Klein).

In 1914, before Einstein had even completed work on his General Theory of Relativity, Nordström struck upon the idea of varying the electrodynamic equations of Special Relativity from one to five while working on his own gravitational theory. His efforts in this direction appeared as "Über die Mòglichkeit, das elecktromagnetische Feld und das Gravitationsfeld zu Vereinigen" in *Physikalische Zeitschrift.*[14] It is important to stress the fact that recognizing Nordström's prior use of what is basically a five-dimensional space-time structure does not imply that he be given any credit as the originator of five-dimensional unified field theories. He merely used the concept to extend Special Relativity, whereas it is common to extend the General Theory of Relativity in unified field theories. Had Nordström continued this line of theoretical work at a later date and attempted a five-dimensional generalization based on General Relativity, then a strong case could be made that he originated this class of unified field theories. But Nordström carried his five-dimensional researches no further than his attempt to extend Special Relativity and thus include gravitation.

Nordström's paper also raises some important questions regarding Einstein's role and opinions in the development of five-dimensional geometries. Einstein had been keeping abreast of Nordström's attempts to explain gravitation and could have been introduced to the five-dimensional concept well before his initial contact with Kaluza by letter. From 1912 to 1914, while Nordström was developing what Einstein later called the first consistent approach to gravitation,[15] Nordström was corresponding with Einstein, and in July of 1913 the two men met in Zurich. In 1914, during the same month that Nordström's five-dimensional treatment of Special Relativity appeared in *Physikalische Zeitschrift*, an article by Einstein and A.D. Fokker, "Die Nordströmische Gravitationstheorie von Standtpunkte des Absoluten Differential Kalkuls," appeared in *Annalen der Physiks*.[16] This article discussed Nordström's gravitation theories in critical detail. It is difficult to believe that Einstein had no knowledge of Nordström's five-dimensional theory at that time, considering how closely Einstein was following Nordström's work, even though he told Kaluza that Kaluza's five-dimensional approach was quite unique and original just a few years later. His comments to Kaluza seem to indicate that Einstein had no prior knowledge of Nordström's five-dimensional paper. Whether Einstein was aware of this paper or not, it must still be remembered that he considered Kaluza's idea as original. This is a further indication of Kaluza's priority regarding the concept of five-dimensional unified field theories.

This new revelation also raises the possibility that Kaluza was aware of Nordström's idea, although this would be very difficult to prove. It would seem likely that Einstein or some other scientist would have brought Nordström's work to Kaluza's attention, had he not had any previous knowledge of it. This fact, if confirmed, would still not detract from Kaluza's original handling of the five-dimensional concept, but would be of historical interest nonetheless. Any relationship between Kaluza's theoretical researches and the earlier concepts of four-dimensional spaces would prove quite interesting, but it would be extremely difficult to demonstrate a direct connection between Kaluza's theory and the pre-Relativistic hyperspace theories.

As was stated, others may well have considered the concept of the fifth dimension shortly after Einstein's General Theory of Relativity was finalized. It could be argued that the idea was "in the air" at the time. The idea of space-time curvature itself, inherent in Einstein's theory, can be thought of as allowing for the continuum to be embedded in a space-time of still higher dimensionality. The fifth dimension was mentioned and considered in a series of articles by Edward Kasner[17] of Columbia University in 1921, shortly before Kaluza's theory was published. Kasner's papers, however, do not deal with a new five-dimensional theory, but instead discuss the possibility of metrical fields that are immersed in a flat space of five dimensions. Kasner even acknowledged the possibility of representing the solar gravitational field in a flat space of six dimensions as well as representing other physical forces in flat spaces of between six and eleven dimensions. As was stated, Kasner was not attempting to develop a unified field theory, but he did make some statements that came very close to the ideas expressed in Kaluza's theory.

Kaluza's considered five space-time coordinates with the value of the fifth coordinate held constant. By comparison, Kasner stated "the existence theorems show that *the solution depends on not more than five arbitrary constants* (one of these is trivial, being a constant factor)."[18] It would seem strange for Kasner to refute the possibility of a five-dimensional flat space and work with space-time continuums of higher dimensions in a scientific atmosphere which was completely anathema to, and unknowing of, such concepts. Still, Kasner made no reference to other papers dealing with higher-dimensional spaces and it is not known if he was either aware of or had any particular five-dimensional theories in mind when he wrote his own papers.

The whole concept of such spaces (or space-times), having dimensions of a higher number than our normal three-dimensional space (or four-dimensional space-time), have their basis in Bernhard Riemann's Habilitation thesis, "On the Hypotheses Which Lie at the Basis of Geometry." In a strict mathematical sense, Riemann established the precedent for spaces of higher dimensions than our normal physical world in the 1850s when he stated that

> **If one regards the variable object instead of the determinable notion of it, this construction may be described as a composition of a variability of n+1 dimensions out of a variability of n dimensions and a variability of one dimension.**[19]

And,

> **Every system of points where the function has a constant value, forms then a continuous manifoldness of fewer dimensions than the given one. These manifoldnesses pass over continuously into one another as the function changes; we may therefore assume that out of one of them the others proceed, and speaking generally this may occur in such a way that each point passes over into a definite point of the other; The cases of exception (the study of which is important) may here be left unconsidered. Hereby the determination of position in the given manifoldness is reduced to a determination of quantity and a determination of position in a manifoldness of less dimensions. It is now easy to show that this manifoldness has n-1 dimensions when the given manifoldness is n-ply extended.**[20]

If these two cases are to hold true in physical reality, then our consideration of a world where position is determined by three spatial dimensions and one temporal dimension can possibly lead to a physical reality which can be described by a five-dimensional (or higher) manifold. Max Jammer, with just this notion in mind, later concluded that "With the rise of Non-Euclidean geometry and other generalizations of classical geometry it became evident that pure mathematics, not logically confined to three dimensions, could operate consistently with concepts of space that possess any arbitrary number of dimensions."[21]

The implications of the new geometry did not escape the notice of physical scientists who were put on the defensive by speculation of higher-dimensional spaces during the nineteenth century and made numerous attempts to prove that the true space of physical reality three-dimensional as it was sensed. William K. Clifford, whose translation of Riemann introduced these new geometrical structures to the English speaking world, carried the mathematical implications of Riemann's geometry into the physical realm and speculated as early as1870 that all matter is nothing but curvature and motion is the ripples of curvature in space and time.[22]

> **We may conceive our space to have everywhere a nearly uniform curvature, but that slight variations may occur from point to point, and themselves vary with time. These variations of the curvature with the time may produce effects which we not unnaturally attribute to physical causes independent of the geometry of our space. We may even go so far as to assign to this variation of curvature *what really happens in that phenomenon which we term the motion of matter.***[23]

By openly stating these opinions and beliefs, Clifford demonstrated his anticipation of both the ideas later found in Einstein's General Theory of Relativity and the various unified field theories, making him the only true forerunner to a whole group of modern scientists and philosophers who consider space-time curvature to be a representation of reality rather than a mathematical expediency.

One possibility that may arise from the acceptance of a real curved space-time is that higher dimensions than our sensible four can exist. This was true in the time of Clifford and Riemann, in the time following the completion of General Relativity when Kaluza put forward his theory and it is still true today. The problem presented by the concept of space-time curvature is still of metaphysical and scientific importance today. The mathematics of General Relativity allows for the reality of the space-time curvature, which in turn may imply a five-dimensional manifold (or higher) in the case of extrinsic curvature. Our four-dimensional world could be embedded in such a higher dimension as opposed to a flat Euclidean space, which would exhibit the characteristic of intrinsic curvature. If this were to prove true, then it would remain for physicists to consider the physical nature of the five-dimensional manifold and develop a proof or physical demonstration of its existence.

The possibilities raised by Riemann's mathematics also gave rise to a school of thought that absolutely refuted that space could be anything but three-dimensional. Foremost among this group was Ernst Mach, the Austrian Physicist and unwitting founder of the logical positivist movement in the philosophy of science. In his book *The Science of Mechanics*, Mach stated

> **As mathematical helps of this kind, spaces of more than three dimensions may be used, as I have elsewhere shown. But it is not necessary to regard these, on this account, as anything more than mental artifices.[24]**

As a footnote to the above comment, Mach went further in his attack on what he believed to be pseudo-scientific theories concerning the higher-dimensional spaces that were developed during the same period of time.

> **We must not hold the mathematicians responsible for the popular absurdities which their investigations have given rise to. ... . The phenomena mentioned were not forthcoming until after the new views were published, and then exhibited in the presence of certain persons at spiritualistic seances. The fourth dimension was a very opportune discovery for the spiritualists and for the theologians who were in a quandary about the location of hell. ... Even the tricks that prestidigitators, in the old days, harmlessly executed in three dimensions, are now invested with a new halo in the fourth. ... We have not yet found an accoucheur who has accomplished parturition through the fourth dimension. If we should, the whole question would at once become a serious one.**
>
> **Everyone is free to set up an opinion and to adduce proofs in support of it. Whether, though, a scientist will find it worth his while to enter into serious investigations of opinions so advanced, is a choice which his conscience and instinct alone can decide. If those things, in the end, should turn out to be true, I shall not be ashamed to be the last to believe them. What I have seen of them was not calculated to make me less skeptical.[25] (The complete text of this quotation is given in the Appendices)**

Some specific points regarding Mach's opinion of the four-dimensional spaces are especially noteworthy:

(1) Even though the fourth dimension was misused by the spiritualists, theologians and prestidigitators, Mach hesitantly believed (or at least so stated) and admitted that science would have to account for this fact if someone were found to actually use a higher-dimensional space to some end.

(2) Mach did not blame the mathematicians for the popular misconceptions dealing with their theories.

(3) Mach admitted that only conscience and instinct could guide a scientist into seriously investigating such phenomena.

(4) He considered higher-dimensional mathematics useful, but only as a mental artifice.

Mach, it seems, was still open-minded enough to accept whatever could be shown to occur in our world, if only begrudgingly. He would only be forced to accept unorthodox and abstract concepts in the face of overwhelming evidence to the contrary. He also unwittingly anticipated some of the problems with later serious attempts to derive scientific theories based on five-dimensional space-times, namely, that such hypothetical frameworks could only be regarded as mental artifices without overwhelming evidence for the reality of a fifth dimension.

Mach gave no indication as to whom he referred as establishing a four-dimensional space, but it can be assumed that he had J.K.F. Zöllner, a German professor of Astronomy, in mind. Although he was an accomplished scientist, Zöllner attempted to use the fourth dimension of space to explain spiritualistic phenomena as well as religious miracles. In his book *Transcendentale Physik*,[26] Zöllner stated that Mach's teachings were congenial to his own theories. Mach, notorious for his belief only in what he could experience or sense, must have been furious with Zöllner's reference to him. Therefore, it can be assumed that Mach's comments were made with Zöllner in mind as well as other scientists who expressed similar ideas. The historical consequences of both Mach's and Zöllner's opinions are well noted, each giving rise to a particular school of thought concerning the physical theories of the present day five-dimensional concept·

Historically, the possibility of physical theories regarding spaces of four dimensions (during the late nineteenth century) or space-times of five dimensions was not unknown, although not well established or even known in general, by the advent of Kaluza's theory. Riemann established a firm mathematical precedent without making any conclusive statement regarding the physical reality of his mathematics. Clifford could be said to have established the tradition of a belief in the reality of space-time curvature, while Mach and others like him established the tradition of those who regard space-time curvature as nothing more than mental artifices. Zöllner and his peers adversely affected the later development of hyper-spatial theories in physics by causing them to be branded pseudo-scientific from the start. These men set the stage onto which Nordström, Einstein, Kasner, Kaluza and others ventured, where five-dimensionality was waiting in the wings for Kaluza to grasp it and use it.

A more direct link between the work of Clifford and the earliest acceptance of General Relativity can be found in the early publications of Sir Arthur Eddington on General Relativity. Eddington's first expositions of General Relativity in the press placed it, quite clearly, into a five-dimensional framework. Although this framework had little or nothing to do with Kaluza's theory, predating Kaluza's published theory by a few years, the existence of Eddington's publications offers food for thought. Eddington clearly owed his five-dimensional worldview to Clifford. However, some points regarding this turn of events bear mentioning. Although the work of Clifford and his followers did not directly influence the researches of either Einstein or Kaluza, the popularity of Clifford's views on the physics of hyperspaces was instrumental in developing the positive scientific attitudes toward hyper-dimensional spaces that aided the rapid acceptance of Einstein's radical theory of General Relativity. In so far as the five-dimensional concepts were 'in the air' when Kaluza's theory was first published, much of the credit for that fact can be traced through Eddington back to Clifford and his followers.

By pointing out and discussing the succession of ideas which culminated in Kaluza's theory and drawing attention to the fact that past and present arguments regarding the reality of space-time curvature are related (at least by a tenuous thread of historical facts) to the five-dimensional space-time structures, it has been demonstrated that it is not an unnatural abstraction to speculate on the physical reality of five-dimensional manifolds. Even so, none of these historical and philosophical meanderings should be expected to detract from the originality and importance of Kaluza's work.

1.3 POST-KALUZAN FIVE-DIMENSIONALITY

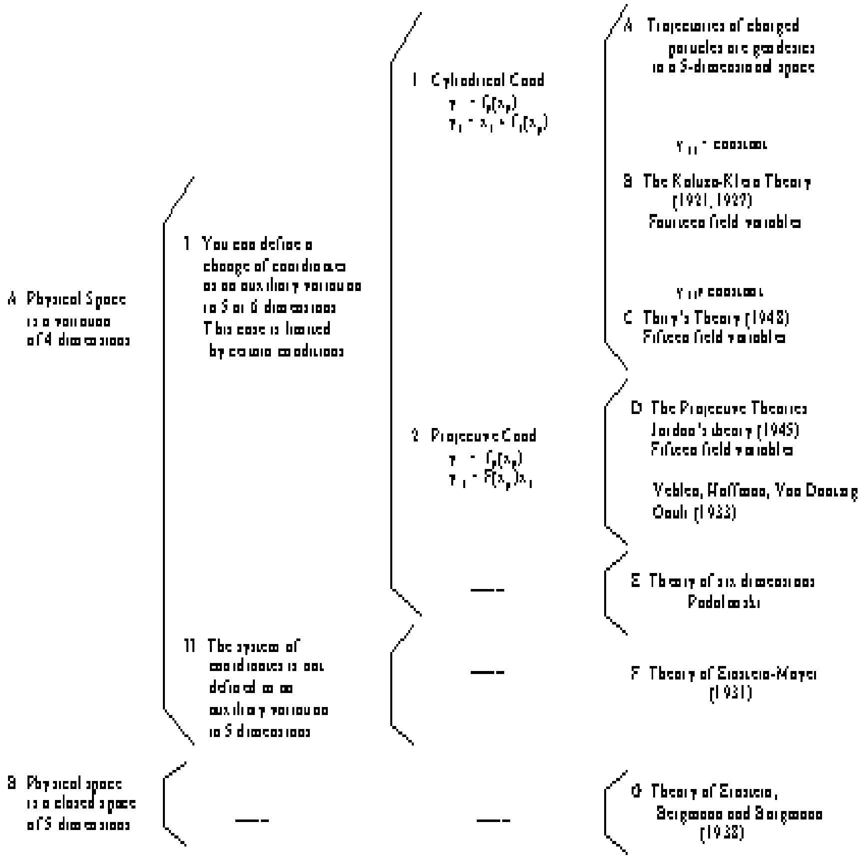
Kaluza's original theory has been extended or otherwise generalized in many different ways, giving rise to different modes of classifying the succeeding theories. Some scientists, while heeding the criticisms of Kaluza's theory, sought more general and thus more acceptable theories by changing one or more of Kaluza's basic assumptions. Others sought to extend the theory into the domain of the quantum and later to include nuclear forces. In this last group of extensions, the non-experiential character of the fifth dimension was seen as a convenient way to include the quantum in the space-time structure. Still others used the five-dimensional (or a higher-dimensional) concept in theories that were only partially related, or wholly unrelated, to Kaluza's theory. These classifications, while important, are based upon physical or mathematical considerations and have less historical significance regarding the development of Kaluza's theory. The historical development of the newer theories suggests a more appropriate way of classifying them.

In the literature regarding the five-dimensional theories there has evolved a specific main line of succession to Kaluza's original work. The theories within this main line of succession can be distinguished as separate from other five-dimensional theories in two different ways. These theories are supported by a specific group of well-known articles and books, including those original to the theories themselves, and within this group of papers the references given are inclusive of other papers inside the group. The few books that deal with five-dimensional theories refer almost exclusively to these papers and no others. When beginning to research the five-dimensional theories, one finds that this group, representing the more easily accessible information, is the first encountered. The group of main-line theories consists of (in addition to Kaluza's theory) the theories of Klein (at least his earlier theories), Einstein and Mayer, Einstein, Bergmann and Bargmann, Jordan and Thiry, Podolanski and the various projective theories.

Of the several books dealing with the five-dimensional unified field theories, one book (in French and never translated) dealing with these theories historically as well as mathematically, develops only the main-line theories. This book, *Les Théories Unitaires de L'Électromagnétisme et de La Gravitation* by Marie Antoinette Tonnelat, is unique in that it is the only book devoted to unified field theories in general.[27] The following is a translation of a chart from Tonnelat's book classifying the five-dimensional theories. It is instructional because it demonstrates the extent of the main-line theories and also serves to compare the partial physical characteristics of these theories.

A Classification of Theories based on the Existence of a Variation with more than Four Dimensions

(Translated from M.A. Tonnelat, Les Théories Unitaires de L'Électromagnétisme et de la Gravitation)



Obviously, there have been very few attempts to discuss the five-dimensional unified field theories within book form; however, a few authors have included sections on these theories within their more complete published discourses on General Relativity. There is the *Théories Relativistes de La Gravitation et de L'Électromagnétisme* by André Lichnerowitz,[28] which deals only with the Jordan-Thiry theory and its relation to the Kaluza theory. Lichnerowitz regards the Jordan-Thiry theory as the most advanced of the five-dimensional theories. In *An Introduction to the Theory of Relativity*, Peter G. Bergmann[29] discusses a general system of mathematics developed for five-dimensional theories while only the theories of Kaluza, Einstein and Mayer, Einstein, Bergmann and Bargmann, Jordan and Thiry and the projective theories are mentioned. *The Theory of Relativity* by Wolfgang Pauli[30] mentions the Kaluza-Klein, Einstein and Mayer, Einstein, Bergmann and Bargmann, Jordan and Thiry theories. *Fields and/or Particles* by D.K. Sen[31] deals only with the Kaluza, Klein, Jordan and Thiry and the projective theories. And *The Conceptual Foundations of Contemporary Relativity* by John C. Graves[32] mentions only Kaluza, Klein and Jordan. It seems quite evident that all of these books stay within what has been delineated as the main line of theories while none of these books offers any significant information on the validity of the concept as a whole.

1.3.1 THE MAIN-LINE THEORIES

The main line of theories began with the work of Oskar Klein in 1926 and 1927.[33] Klein has been credited with both the formalization of Kaluza's original theory and several attempts to extend the theory into the domain of the quantum. For this reason, some scientists and authors have given Klein partial credit for the original theory and refer to it as the Kaluza-Klein theory. Kaluza's theory also seems to have lain fairly dormant until Klein's first exposition of it appeared, thus it may be speculated that Klein was instrumental in popularizing the theory.

Klein saw within the five-dimensional hypothesis a vehicle for introducing the quantum into the space-time continuum rather naturally, as well as a way of accounting for the atomicity of electric charge. He first equated the geodesic in the fifth dimension to the periodicity of the electric potential ϕ. This led to a quantum of action, while a conjugate momentum in the fifth dimension was fixed to account for the positive and negative electrical charges. By forming the five-dimensional Lagrangian of a particle in a combined electromagnetic and gravitational field, and then differentiating it with respect to the velocity along the fifth component, he established a relationship within the field yielding the charge-to-mass ratio. This allowed the conjugate of the fifth coordinate to appear in a manner analogous to the manner by which matter and momentum were conjugates in our normal four-dimensional space-time. The periodicity that he introduced into the fifth dimension also allowed him to make an association between a function in the fifth dimension and Schrödinger's function. Within this theory, there also emerged a fundamental length of $l_o=(2k)(hc/e)$, where k is Einstein's gravitational constant, and h, c and e are Planck's constant, the velocity of light and the electron's charge. Klein later proposed "to relate the fifteenth quantity γ_{oo} with the wave function, which characterizes matter, in order to achieve a formal unity between matter and field,"[34] and thus further include Schrödinger's wave mechanics into the five-dimensional framework. These theories, as well as Klein's formalization of Kaluza's theory, are mentioned quite often in the literature dealing with the main-line theories. However, Klein later developed[35] other versions of his new theory, which are rarely mentioned within the main line of literature on the subject.

Klein admitted that his first attempts were unsatisfactory and for the next decade published nothing more dealing with them.[36] In 1939, he made a new attempt to extend his earlier theories of a grand unification of quantum and field theories by adding the "mesotonic" forces[37] of Yukawa to his theory. Klein thought that the Yukawa potential, in its mathematical treatment, was analogous to his earlier treatment of the five-dimensional framework. Regarding this analogy he wrote that "The direct and general way it expresses the fundamental conservation and invariance theorems seems to make this representation a natural starting point for a general quantum theory comprising also the charged fields, which are supposed to correspond to the mesotons."[38] This theory allowed the construction of a new Lagrangian containing, in addition to the gravitational and electromagnetic components, the spinor as a tensor field component. By using a variational principle, actions between protons, neutrons and electrons, explained by the interactions of neutrinos such as theorized in Yukawa's theory of nuclear forces, could be found. Later, in 1947, the theory was again extended; field equations were found and simplified yielding equations for free mesons while the wave equation for nucleons was derived. To achieve this end, Klein had to replace the original assumption that the field quantities were independent of the fifth coordinate by an assumption that they were periodic functions of x^0 with a period of l_0. This change also introduced an indeterminacy "which would exclude the use of the fifth dimension in any geometrical sense, and has the practical meaning that particles of given charges have naturally coherent wave functions, as is always assumed."[39] A new fundamental length was introduced which was equal to $l_0(e^2/hc)$, where l_0 was the original fundamental length expressed in Klein's earlier work.

Klein made one last attempt to include nuclear forces in his five-dimensional framework. He was dissatisfied with his 1947 attempt in that it had "such features that it should hardly be taken literally."[40] So he derived, via the same periodicity function, "a theory of more physical aspect, whereby charge invariance appears as a part of a natural generalization of gauge invariance."[41] As a further consequence of his concept of a fundamental length, l_o, Klein calculated that a particle, approximating a quantum in a linear wave equation and having a wavelength approaching zero, would have a gravitational self energy approaching the kinetic energy corresponding to its volume. He hoped to do away with the remaining divergencies of the electron theory in this manner. This new and more stringent generalization of the theory from the quantum theory point of view also led to a suggestion of possible states with a multiple charge. Klein's total work could best be characterized as a continuing attempt to save or retain the basic tenets of Kaluza's five-dimensional space-time framework while keeping pace with the advances being made in subatomic physics.

The next major five-dimensional theory was, in contrast to an extension of the type which Klein had made, an attempt by Einstein and Mayer (1931)[42] to retain the four-dimensionality of space-time while using the mathematical framework of Kaluza's theory to incorporate electromagnetism into the field. To accomplish this they introduced a theory using a five-dimensional tensor calculus without introducing either a five-dimensional structure or a five-dimensional coordinate system as Kaluza had done. The tensors used to accomplish this were functions of four coordinates only, even though they used indices varying from one to five. This allowed a two-fold space-time whereby "At each point of the Riemannian space the tangent four-dimensional flat space is assumed to be immersed in a flat five-dimensional space, but the Riemannian space itself is not supposed to be immersed in another five-dimensional Riemannian space."[43] Within this space structure, two parallel displacements of a vector were possible according to special defining conditions. One displacement corresponded to the normal displacement of a vector in four-dimensional space while the other was defined as a displacement with respect to a five-dimensional space. An arbitrary second rank tensor of the field emerged from the mathematics. It was equated with the electromagnetic force. Equations of motion of charged particles yielded the field equations. This theory only allowed for the determination of the field equations in a space occupied by an electromagnetic field. In a second paper this oversight was corrected, in that it became possible to derive the field equations in spaces occupied by distributions of electric charge and current as well.

In 1930, Oswald Veblen and Banesh Hoffman had also sought to explain away the fifth dimension using a projective theory.[44] Important work was done along this same line of reasoning during the early 1930's by Pauli, Van Dantzig, Schouten and others.[45] In these projective theories the fifth coordinate was only considered to be a projection of the four coordinates of a real space-time. The fifth dimension was shown to be merely mathematical in such a way that the basic problem of explaining why we do not sense the fifth dimension was circumvented. These projective structures thus retained the intrinsic nature of space-time while still utilizing the mathematical advantages of the five-dimensional manifold. These formulations were actually no more general than Kaluza's theory and enjoyed a one-to-one correspondence with it, so it was easy to pass from one to the other.[46]

The next attempted modification in the main line of succession came in 1938 by Einstein and Bergmann. [47] In an attempt to give the fifth component some real physical significance, Kaluza's theory was subjected to a fundamental change by the substitution of a closed (or periodic) space in the direction of the fifth coordinate. This change saved the metric by doing away with Kaluza's cylindrical condition. In this framework a geodesic passing through one point along the fifth dimension must then pass through all corresponding points in the fifth dimension. The lines thus formed corresponded to Kaluza's A-lines. In this manner, the space closed on itself and remained free of singularities. A special coordinate system was established whereby a normalized distance of one separated corresponding points on any five-dimensional line. In this theory, the purely five-dimensional component γ_{oo} was the same as in Kaluza's theory, but there was also a corresponding four-dimensional metrical tensor g_{ab} differing only in that "Its components are, however, in general periodic functions of x^0."[48]

The most general field equations possible were generated from this space-time structure according to two conditions: "(1) The field equations should be derived from a variational principle; And (2) The action function should consist exclusively of terms containing either second derivative linearly or products of two derivatives of the second order."[49] These conditions simplified Kaluza's basic assumptions while assigning to the fifth dimension the desired physical meaning, not merely using it as a mathematical formality. This theory was shown to reduce to Kaluza's in the event that $g_{mn,0} = 0$, since g_{mn} is dependent on x^0 (a fact that gave rise to the four vector current density) in this new theory. Thus, this theory is a generalization of Kaluza's theory as well as a simplification.

In the Einstein-Bergman theory, the four field equations representing electromagnetism were integro-differential equations, while the remaining ten equations representing the gravitational field were differential. Einstein, Bergmann and V. Bargmann corrected this difficulty in an additional paper in 1941.[50] The basic geometrical structure of this space-time was retained from the earlier theory, while a system of field equations was derived consisting of only differential equations. In this system there were 14+4 differential equations to represent only fourteen variables, so four identities were introduced to reduce the independent equations to fourteen, the proper number needed to satisfy the fourteen independent variables of a combined electromagnetic and gravitational field. These were shown to provide a uniquely determined system by the covariance and homogeneity conditions while introducing no arbitrary constants. However, this system raised other problems:

> **... that the equations are uniquely determined causes serious difficulties for the physical interpretation of the theory. It seems impossible to describe particles by non-singular solutions of the field equations. As no arbitrary constants occur in the equations, the theory would lead to electromagnetic and gravitational fields of the same order of magnitude. Therefore, one would be unable to explain the empirical fact that the electrostatic force between two particles is so much stronger than the gravitational force. This means that a consistent theory of matter could not be based on these equations.[51]**

It was at this point in time, or shortly thereafter, that Einstein left behind all attempts to establish a unified field theory based on a five-dimensional assumption and returned instead to his non-symmetric derivation.

The theory of Pasqual Jordan and Yves Thiry, next in the main line of succession, actually represents two different mathematical approaches arriving at the same endpoint. In both cases, the constancy of the purely five-dimensional component was sacrificed while the metric was retained.[52] Thiry, in 1946, used a non-projective mathematical formalism based on a simple extension of the five-dimensional theory for his derivation. A normal metric in the four-dimensional space-time has a signature of ---+ (or +++-), while the metric of five-dimensional structure conforms to a hyperbolic space-time structure with a signature of ----+ (or ++++-). Thiry's method, following Eli Cartan's exterior differential calculus,[53] consisted of decomposing the five-dimensional metric represented by an elliptic space-time structure with the signature ----- (or +++++), corresponding to the definition of a moving-frame at each point of the variation. The components of both the Ricci tensor and the energy-stress (matter) tensor were calculated in this framework before passing into an orthonormal and natural framework with a hyperbolic structure and its corresponding signature of ---+- (or +++-+).[54] This method permitted the fifteen field variables to be expressed in such a way that the fifteenth variable need not be constant.

Jordan had used a projective theory in 1945 in which the fifteenth variable was used to represent a scalar field independent of the fifth dimension. The resulting theory was the same as that derived by Thiry, allowed the two theories to be combined as the Jordan-Thiry theory. Jordan further found that the existence of an electromagnetic field implied a variable k, the gravitational constant, when k was associated with the newly varying five-dimensional scalar. The idea of allowing the gravitational constant to vary had been suggested earlier by Dirac.[55] In both systems of mathematics, the field equations reduce to those of General Relativity when k is made constant. Within the context of this theory, new consequences arose for the five-dimensional theories: The weak variations of k were the functions of the ratio e/m and a fifteenth equation arose, depending on the variable k, which implied that the presence of uncharged matter in motion could produce a magnetic field. The second consequence led to an explanation of the Blackett effect that predicted a magnetic field for uncharged rotating matter. Both consequences gave rise to cosmological effects that were later investigated by Jordan and others.

The final theory within the main-line sequence was proposed by J. Podolanski in 1949.[56] Podolanski chose a six-dimensional framework, since the complete set of Dirac matrices represents the $({}^{6}{}_{2})$ rotations in a six-dimensional space. In this system, the four dimensions of the space-time continuum represent a unique subspace while gravitation and electromagnetism both became geometrized inertial forces. Such a theory is considered to be an embedding theory. By adopting this concept, Podolanski was able to do away with the five-dimensional interpretation, as well as Kaluza's condition of homogeneity (as he called the cylindrical condition). As a restriction meant to decompose the six-dimensional space of four dimensions, he imposed a "structure axiom" such that "space admits of a two-dimensional translation."[57] The six-dimensional structure was then, according to this axiom, to be made of laminated sheets. It "folds up into a four parametric family of two-dimensional sheets."[58] The group of points of which a sheet was made was physically indistinguishable, such that "each sheet corresponds to <u>one</u> point in the space-time world and physical phenomena take place in the four-dimensional spaces normal to the sheets."[59] The actual construction of this space from the laminated sheets was governed by the field equations. The field equations determined how the electromagnetic sources were affected by the structure and how the structure further affected the electromagnetic sources. The electromagnetic forces satisfying the Maxwell equations emerged when the four-dimensional space-time was embedded in the six-dimensional structure in such a way that two fields of inertial forces, having the character of forces of constraint, resulted. The connection imposed by the six-dimensional space-time on the four-dimensional space-time caused a deviation from the Riemannian metric, which allowed an expression of the 'forces of constraint.' This theory tended to specialize the Riemannian metric in the manner prescribed rather than generalize it, as had been the case in other theories.

All of these main line theories have enjoyed various degrees of notoriety. Anyone researching the five-dimensional theories will find many references to them. However, there are a number of other theories and modifications based on Kaluza's theory or similar five-dimensional (or higher) structures that are not as well known and have gone without the acknowledgment enjoyed by the main-line theories. This is not to say that these theories are not without merit, nor is it to say that they deserve more recognition than they have received in the past. It is only to say that they should be recognized as legitimate attempts, and probably valuable attempts, to describe our physical world.

1.3.2 THE OFF-LINE THEORIES

The work of H.T. Flint, beginning in 1928,[60] offered what is undoubtedly the longest (three decades) and most sustained attempt to develop a modification of Kaluza's theory. His efforts centered about a grand unification between field theory and the quantum theory. The many publications of Flint's can easily be identified as a continuous development and amplification of his earlier ideas, rather than a series of changing ideas concerning the fifth dimension. In this respect, his work parallels Oskar Klein's both in the duration of his efforts and in the similarities between the finer points of their theories as well as the complete theories. In his earliest work, he attempted to incorporate quantum ideas directly into a space-time framework using Weyl's and Eddington's concept of parallel displacement of a vector (1927 in collaboration with J.W. Fisher).[61] However, Flint soon adopted the five-dimensional approach of Kaluza's theory in hope of overcoming the difficulties of his earlier work. He continued this line of theoretical work until the late 1950s.

Flint eventually adopted a concept using the notion of the matrix length of a vector, which he treated as a distance under parallel displacement within the five-dimensional field, much as Weyl and Eddington treated the parallel displacement of a vector in a four-dimensional continuum. With this method, Flint was able to derive first order quantum equations in 1935. However, he was unsatisfied with the way in which the equations entered into the mathematics instead of being derived directly from the mathematics of the field. Subsequently, this development was continued and refined in later work by Flint and the method of a five-dimensional displacement became a characteristic of his later derivations.

Other characteristics of Flint's work include:

(1) The use of an operator in the form of a partial differentiation with respect to the fifth coordinate, such that the operation on any function is the same as multiplying that function by $2i(mc/h)$. This is equivalent to the association of the fifth component of momentum of a test particle with the scalar quantity mc.

(2) Fundamental lengths of h/m_oc (the Compton wavelength) and e^2/m_oc^2 were used.

(3) A 'principle of minimum proper time' was derived, giving a smallest detectable length of $l = (h/m_oc)B(1-B^2)$ as well as a smallest detectable time of $t = (h/m_oc^2)[l/(1-B^2)]$ where $B = v/c$. For small distances, this principle corresponded to the Heisenberg Uncertainty Principle.

Flint eventually derived first and second order quantum equations as well as an equation analogous to Schrödinger's equation directly from the field structure. He was also able to explain the quantization of charge and mass. In 1944,[62] he applied his concept to the meson theory and a year later[63] attempted to explain nuclear fields in a similar manner. In these applications of his basic five-dimensional hypothesis, Flint presented the notion that "the equations of the quantum theory are gauging equations in a geometrical and metrical system suited to the world of physics."[64] He had found in Kaluza's framework an appropriate space-time structure on which to base his own system as well as a convenient way of explaining the 'mc' term without introducing it from outside of the system. The 'mc' term corresponded to a five-dimensional momentum.

Flint had found it necessary to make changes in Kaluza's original formulation in order to accomplish his own goals. Aside from the above-mentioned modifications, Flint changed Kaluza's space-time structure in order to avoid the earlier criticisms of Kaluza's work. He noted that the more serious objections presented were that the "general covariance is destroyed by the 'cylindrical' condition, that the g_{μ} do not contain x^5 and that g_{55} is taken as constant."[65] He overcame these difficulties by "regarding the special use of the cylindrical condition and the assumption about the way x^5 occurs in the functions as an approximation required by our need to eliminate x^5, in interpreting our results in the light of our present knowledge of physical phenomena."[66] It almost sounds as if Flint was making a qualifying statement regarding the reality of the fifth dimension.

During his development of the five-dimensional concept of the quantum theory, Flint proceeded as if there were some reality to the concept, but he made no statements which demonstrated his belief in the reality of a fifth dimension. It is hard to comprehend the fact that Flint did not believe in the reality of the fifth dimension when so many qualities were attributed to it. His one statement regarding this subject only referred indirectly to the reality issue. His ambivalence in publicly supporting the reality of a fifth dimension is further evident in statements that would seem to indicate caution (at least) in granting some essence of reality to the fifth dimension, even while he publicly put forward a face displaying a safe disregard for its reality.[67]

William Wilson, an associate of Flint's, also attempted to merge quantum theory and relativity within the five-dimensional field. In 1922,[68] he published an article discussing the relation of quantum theory and electromagnetism, while in 1926[69] he extended his discussion by the addition of a five-dimensional framework. Although he gave no credit for the basis of this framework to Kaluza, he did state in a footnote that Flint had pointed out to him that his ideas "were exactly" similar to those found in O. Klein's paper of 1928.[70] In his later paper, Wilson derived an equation which became identical to Schrödinger's equation in quantum mechanics upon a simple substitution. The difference between these two equations being that Wilson used the concept of a 'Volume' in five-dimensions, whereas Schrödinger's Ψ function later became associated with and/or equated to a probability density. Wilson continued his development of this equation deriving a second equation, which he showed to be equivalent to Schrödinger's equation for the Hydrogen atom under a proper choice of limits. From these derivations Wilson was able to define his 'Volume' as follows:

> **If a particle at some instant is actually within a 'volume' V_o it will be within a volume V, Which is the parallel displacement of V_o, at some time later (or earlier) instant. If its position at any time is unknown, the probability that it is in a specified volume will depend in some way on V. This is, in fact, the usual meaning of V or Ψ.[71]**

Thus, the correlation between Schrödinger's equation and those derived by Wilson seemed complete for all intents and purposes.

In another article,[72] written in collaboration with Miss J. Cattermole in 1938, Wilson derived the quadratic operator of Special Relativity, $p_x^2 + p_y^2 + p_z^2 - m_x^2c^2 + m_0^2c^2 = 0$, by using a five-dimensional representation of Special Relativity. Using linear operators within the context of five dimensions, Wilson and Cattermole showed that this operator was equivalent to Schrödinger's equation,

$$\Delta^2 \Psi - 1/c^2(\delta^2 \Psi/ \delta t^2) - (4 \pi^2 m_o^2 c^2/h^2) \Psi = 0 .$$

During this period (or perhaps shortly before), Wilson[73] also collaborated with Flint in his work with the five-dimensional concept. Further, Wilson has the distinction of being (probably) the only scientist to publish a statement clearly supporting five-dimensional theories. According to Wilson,

> **Einstein himself described a unitary theory of great interest; but this too does not seem to furnish an acceptable solution of the problem f making electromagnetic phenomena an organic outcome of the geometrical properties of the continuum. The most attractive and probably the correct solution is one which has been developed by Kaluza and others.[74]**

This statement of strong support, made during a time when no one else seems to have been willing to make such a statement, appeared in no article or paper on those theories but in a book by Wilson on theoretical physics.

J.W. Fisher also worked with the five-dimensional framework during this same period, both in collaboration with Flint and alone. He was able to derive an analogy between the wave equations of light in the space-time continuum and the wave equation for a particle in the fifth dimension.[75] In the five space advocated by Fisher, everything became a radiation problem. In this manner, all particles were shown to travel null geodesics in the continuum. Flint and Wilson later used this idea in separate developments. Fisher's early collaboration with Flint was also helpful in establishing the five-dimensional formalisms that Flint was to use throughout his career.[76]

Another important contribution to the field of science bears mention at this point, although it would seem to have little to do with Kaluza's theory or its later development. This point is the Fundamental Theory of Sir Arthur Eddington. Eddington was one of the earliest proponents of General Relativity and led the solar eclipse expedition that first proved that light rays bend confirming Einstein's prediction. During his career, Eddington worked fervently on the affine connection developed by Weyl and otherwise continued his theoretical work on relativity. However, Eddington's primary worldview was that of a five-dimensional framework well before Kaluza's theory was published. Eddington's attitudes were directly affected by W.K. Clifford, whose work influenced all of Eddington's research and philosophical outlook on physics as a whole. After Eddington died in 1946, a number of his essays, talks and publications were published as *The Fundamental Theory*. This theory utilized a three-dimensional space and a two-dimensional time. This structure was not based on Kaluza's theory, but used a five-dimensional system based on Clifford algebra. Eddington's use of a five-dimensional space-time system and his philosophical attitudes clearly demonstrate the extent to which such ideas were 'in the air' during the developmental stages of Kaluza's theory even though Eddington's work had no direct connection to Kaluza's research or theory.

In 1934, D. Meksyn took a different approach and developed a very specialized theory with an eight-dimensional unified field. This theory dealt with a two particle system whose space metric satisfied Einstein's law, $G_{\mu} = 0$. Of the two articles[77] in which the theory was developed, the first took into account only the cases of the electrostatic and magnetic fields. Meksyn found that the "electrostatic field is simply connected with the fundamental tensor, but the vector potential and the electromagnetic force have no metrical meaning"[78] while "the law of motion and the connection between the two times is given by the geodesic of space."[79] When the gravitational case was employed in his second paper, Meksyn further found that the classical case for two particles was duplicated and the concept of center-of-mass held true.

In yet another attempt, G. Vranceanu[80] changed Kaluza's space-time structure and developed a theory around a non-holonomous hypersurface, $V^4{}_5$, embedded in a five-dimensional Riemannian space. In such a space, it was hypothesized that a point in space-time was locally four-dimensional, but the parallel displacement of a vector over any closed circuit wouldn't necessarily have returned the vector to its starting point, but to some point along the normal to the local space-time of the starting point in the fifth dimension. The tensor describing the four-dimensional behavior corresponded to the curvature tensor in General Relativity, although a torsion-tensor was introduced representing the displacement along the normal. This tensor was identified with the electromagnetic tensor. The paths of charged particles moving in a combined electromagnetic and gravitational field could then be deduced. This theory was advanced still further by Kentano Yano,[81] who first compared it to other five-dimensional theories, elaborated the mathematics and then derived the mathematical conditions which had been assumed by Vranceanu.

J.G. Bennett, R.L. Brown and M.W. Thring[82] developed a theory in 1949 that was unique with respect to all other five-dimensional theories in that it differed in its basic concept, having nothing to do with a space metric. In their estimation, "We have endeavored to show that a consistent and fruitful world picture is obtained by extending the space-time framework to a five-dimensional scheme free from the complications of a Riemannian or affine geometry."[83] The framework established was an extension of Minkowski's "absolute world" accomplished by the addition of a fifth orthogonal direction labeled "anti-time" or "eternity." Fields were then identified "with the manner in which the four way measuring system of the observer O is embedded in a flat five-dimensional manifold."[84] This system depended on a notion of "time-blindness" regarding the fifth-dimensional or "anti-time" component. An absolute straight line called a "cosmodesic" was used to describe all unconstrained motion in which the particle and the absolute reference frame, in which the particle was measured, are free of curvature. This presented a "simple and natural extension of Newton's first law."[85] "True" intervals in the manifold corresponded to a line element of,

$$(RS)^2 = -({}^1Q)^2 - ({}^2Q)^2 - ({}^3Q)^2 - ({}^4Q)^2 + ({}^5Q)^2,$$

where the Q's represent the coordinates in each of the five dimensions and "RS" represented the distance interval.

Eventually, it was found that the "field theory becomes then the science of the relations between simple unconstrained bodies moving in cosmodesics and space extended rigid systems used by physical observers for making measurements."[86] Any physical observer would wrongly conclude that the space-time continuum had curvature upon another object's cosmodesic when applying a variational principle in his own four-coordinate system. This action also had the consequence of introducing two distinct components or accelerations, one corresponding to gravitation and the other to electromagnetism. This theory seemed to represent more of a perversion of the concept of space-time curvature and an attempt to save Newtonian mechanics in the face of General Relativity and the unified field theories, than to attempt a further generalization to find a complete field structure.

In 1945, H.C. Corben[87] introduced his own attempt to treat gravitation and electromagnetism in a unified manner, following Nordström's 1914 suggestion, by extending the framework of Special Relativity to five dimensions. This was accomplished by varying the Maxwell-Lorentz equations and the Lorentz condition[88] over the values of one to five, rather than the normal one to four of space-time. In this approach, Corben also thought it important to set the partial derivative with respect to the fifth coordinate equal to zero, which was the same as assuming the flatness of space-time, and thus disregarding the metric as found in the General Theory of Relativity. Because of this condition, he stated that his theory should be treated with caution.[89] A new group of equations was found to emerge, whereby new quantities acting like the rest density and gravitational potential were found. The theoretical framework also yielded results predicting that gravitational waves would be propagated with a finite velocity equal to that of light, gravitational attraction would only act on the rest mass of a body regardless of the relative velocities of the gravitating bodies, and an "accelerated mass moving in a vacuum emits energy which at large distances assumes the form of longitudinal waves which have maximum intensity in the direction of the acceleration."[90]

In this first attempt by Corben, the extra dimension was considered to have no physical significance.

> **We cannot expect to see a physical significance which can be attached to the t' within the framework of the special theory, since as then nothing changes with t' it has NO physical significance.[91]**

However, in his second paper,[92] Corben postulated that the five-dimensional continuum was flat, but "in common with the theories mentioned above (Kaluza, Klein, Pauli, Einstein, Möller, Wilson and Cattermole), I postulate that the fifth coordinate is spacelike."[93] Further, calculations were then made using Lagrangian densities and the resulting equations, under proper conditions, allowed for the creation and annihilation of matter, which would seem to be an appearance or disappearance into the fifth dimension. It was further shown that a charged particle moves along a null geodesic in the five-dimensional continuum, allowing the extra dimension to be equated to the proper time. From this, a corresponding uncertainty principle, $\Delta k \; \Delta s \geq 1$, was derived. As the rest mass, Δk, and the proper time, Δs, appear as conjugate variables, a unified vector-pseudoscalar theory of mesons also appeared as a special case of the fields in Corben's consideration.

Corben returned with a new five-dimensional field theory several years later, in 1952,[94] but this time he dealt directly with the Kaluza theory instead of the Special Theory of Relativity. Kaluza's constant "A" was chosen in such a way that it led to the correct mass difference between a proton and neutron, allowing these particles to be treated as different states of the same particle. The electric charge was then assumed to be dependent on the extra degree of freedom. The field equations and geodesics of particles yielded a possibility that nucleons may interchange their charges and a photon, when within a very short range of a charged particle, might acquire both a charge and a rest mass.

During this same period, 1946 to be more exact, K.C. Wang and K.C. Chang[95] of the National University of Chekiang in China published still another hyperspace theory. In their theory, it was assumed that there were four space-like dimensions and one time-like dimension. Both momentum and velocity in the fifth dimension were equal to zero and the density distribution along the fifth dimension varied as the cosine $(kx_o+ \varepsilon)$ or sine $(kx_o+ \varepsilon)$, where k and were both arbitrary constants. This last assumption was equivalent to saying that a particle could be represented as a line extending in the fifth dimension while the density along that line varied as a function of the cosine in the fifth dimension.

From these basic assumptions, Wang and Cheng developed a theory that agreed with the electrodynamics of classical theory. When applied to the meson theory, they were also able to obtain the vector, pseudovector and couplings with a restriction that all of the couplings must be of equal importance. Their theory corresponded to the weak coupling found in other current theories in quantum mechanics such as Pauli and Kusaki's theory.[96]

One last theory[97] bears mentioning, although not because of its method of derivation or its consequences since it is virtually the same as Jordan's theory. Instead, the existence of this theory raises a particular historical point of interest. Einstein and Bergmann developed a theory in 1938 equivalent to Jordan's later theory. At the time of that development, they didn't think well enough of their theory to publish it. According to Bergmann's later testimony,

> **In the spring of 1946, Professor W. Pauli turned over to the author of this paper galleys of a paper by P. Jordan entitled 'Gravitationstheorie nit veränderlicher Gravitationszahl', which was to have appeared in the Physikalische Zeitschrift sometime in 1945, but which was, of course, never published because Phys. Zeitschrift in the meantime ceased publication. In this paper Jordan attempted to generalize Kaluza's five-dimensional unified field theory by retaining g_{55} as a fifteenth variable. Professor Einstein and the present author had worked on the same idea several years earlier, but had rejected it and not published that abortive attempt. The fact that another worker in this field has proposed the same idea, and independently, is an indication of its inherent plausibility. Therefore, it seemed worthwhile to review these attempts to 'vary the constant of gravitation' and to discuss the possibilities inherent in geometries of this kind.[98]**

Bergmann did publish the theory, but not until after Jordan published his own version of the theory. This incident raises important questions regarding the development of five-dimensional theories after Kaluza. Did Einstein, either independently or in collaboration with others, develop other unknown five-dimensional theories? Could other scientists have done likewise? Why should an atmosphere exist where scientists do not publish their theories? Do other theories authored by other scientists exist, buried deep in the historical records, which are still to be found?

Regarding the first question, it can only be stated that Einstein was known to have tried many different ways of unifying electromagnetism and gravitation, but only to have published a few of these attempts. It would be easy to conclude that he may very well have made other attempts regarding the five-dimensional structure, but never published these because he was dissatisfied with them. Einstein was very cautious regarding his opinion of the five-dimensional theories and it is easy to speculate that this caution would have kept him from publishing some of his five-dimensional ideas. Only a thorough search of his personal papers could bear any light on this matter. It is also very possible that other scientists have been confronted with the same problem and kept their speculations and ideas to themselves. A major part of this thesis is dedicated to showing that the five-dimensional hypothesis is one of several natural extensions of the General Theory of Relativity, that this idea was 'in the air' and that the scientific community's general attitude toward theories of this type made it difficult to develop them. If these assumptions have any truth to them, then it is very possible and even highly likely that other scientists were so cautious in their speculations and/or more serious theoretical researches on five-dimensional space-time structures that they did not publish or otherwise publicize their ideas and views on the subject.

The idea of caution, when dealing with radical scientific ideas, is not new in the history of science and has a very real and significant (in the case of five-dimensional theories) parallel in the past. The mathematics of General Relativity and thus Kaluza's theory are based upon Riemannian geometry. Bernhard Riemann was a student of J.K.F. Gauss. Gauss had a very strong influence on Riemann's original development of these geometries since it was Gauss who chose the subject for the 'Habilitation' thesis, which became the basis of Riemann's geometry. Historically, the class of Non-Euclidean geometries of which Riemann's is only one example was co-founded by Wolfgang Bolyai and Nicholai Lobachewski. Bolyai sent a letter to Gauss in the early 1830s informing

Gauss of his discovery of a non-Euclidean geometry. Gauss replied that he had already made that discovery nearly thirty years before, only informing a few friends of his discovery. The reply angered Bolyai who did not know that Lobachewski had discovered and publicized the new geometry several years earlier.

Yet there exists a great deal of evidence that Gauss was the first to discover the non-Euclidean geometries, but hid his discovery allowing Bolyai and Lobachewski to rediscover the new geometries independent of Gauss' work. The caution that Gauss demonstrated by not revealing his important discovery to a skeptical academic community is still present today regarding radical changes in our present understanding of physical reality, such as the five-dimensional unified field theories would make. The significance of this particular historical parallel is easy to see. The attitude that Mach and others portrayed in rejecting the earlier Riemannian geometries, as a basis of physical reality is virtually the same attitude that rejects the possibility of five-dimensional theories today.

The last question, dealing with the derivation of equivalent theories independent of each other, can be answered more easily in the affirmative. Wilson's theory of 1928 closely paralleled Klein's theory of 1926, while Flint's theory of 1946 closely paralleled Klein's theory of 1946, both dealing with the application of five-dimensionality to the meson and nuclear theories. Both Wilson and Klein (in 1946) were informed of the similarities of their respective theories to the other's work and commented on them in footnotes to their articles. Wilson[99] assured his readers that his and Klein's ideas were arrived at independently and Klein promised to give the matter of Flint's theory more attention at a later time.[100] The phenomenon of parallel development of ideas is common in the history of science. The case of Gauss, Bolyai, Lobachewski and Riemann, stated above, is only one of many such examples. Such parallels are at times used to illustrate that certain ideas occur to different people in different places when science has reached a point where the ideas represented in the new theory are ready to be developed. The ideas are more-or-less "in the air" waiting to be developed. If this view is applied to the case of five-dimensional theories, both at the time of Kaluza, Kasner and Nordström, and at later times concerning the extensions of Kaluza's theory, a case presents itself whereby the five-dimensional concept may not be that radical of an idea and was thus considered by some scientists to be the best logical extension of General Relativity. It must be remembered that the idea of a real four-dimensional space with a separate time had been an issue in some mathematical, academic and scientific circles for the last three decades of the nineteenth century. So the concept of a five-dimensional space-time should have been a natural result of scientific speculation after the initial development of General Relativity. It is to Kaluza's everlasting credit that he carried the idea beyond the stage of mere speculation and developed a working theory.

Another feature dealing with the historical development of these theories also has become evident. It has sometimes happened in the history of science that ideas and concepts have developed around opposing schools representing different basic concepts or theories as well as different geographical locations. An example of this might be seen in the English and European schools of thought regarding the concepts of absolute versus relative space, as expressed by Newton and Leibniz. In the development of the five-dimensional theories this also seems to have taken place to a small extent. Specific schools of thought concerned with the modification and extension of Kaluza's theory seem to have been centered in London and Paris. These are not absolutely opposed schools of thought, but their existence demonstrates the tendencies toward different modes of development of the theory. The French school seems to be centered more around the ideas of Thiry, Tonnelat, Lichnerowitz, Jordan and at a later date Souriau and Costa de Beauregard, while the English school is centered about Fisher, Flint, Wilson, Cattermole and at a later date, Williamson.

These schools can also be portrayed to a small degree by certain characteristics, as follows:

(1) The French theorists tended to regard more basic changes in the Kaluza theory and the development of new alternative theories whereas the English school tended more toward extending the theory as it stands.

(2) The French school tended to be more abstract, elegant and grander in their concepts while the English school tended to be more practical and application conscious.

(3) The French tended more toward the main line of succession while the English school seems to have followed a course more independent of the main line.

These are merely generalizations and there are several theories that cross from one school to the other as well as some theories and individuals that belong in neither group. An interesting case deals with Klein's theories. The main-line articles tend to mention only Klein's articles of 1926 and 1927, possibly because it was in these articles that Kaluza's theory reached its final formalization. Klein's later work can then be characterized as outside of the main line of theories. However, his mathematical applications, especially in his later theories, closely parallel the work done within the English school. Klein could therefore be classified as belonging to the English school according to his later work on the five-dimensional hypothesis. On the other hand, Podolanski worked in England yet his theories are main line and he would be better suited in the French school. The development of the theories along these different lines offers some interesting speculation, but the implications of this line of reasoning are not especially germane to this study will not be followed any further at this time.

1.4 A BRIEF SUMMARY OF THE HISTORICAL DEVELOPMENT

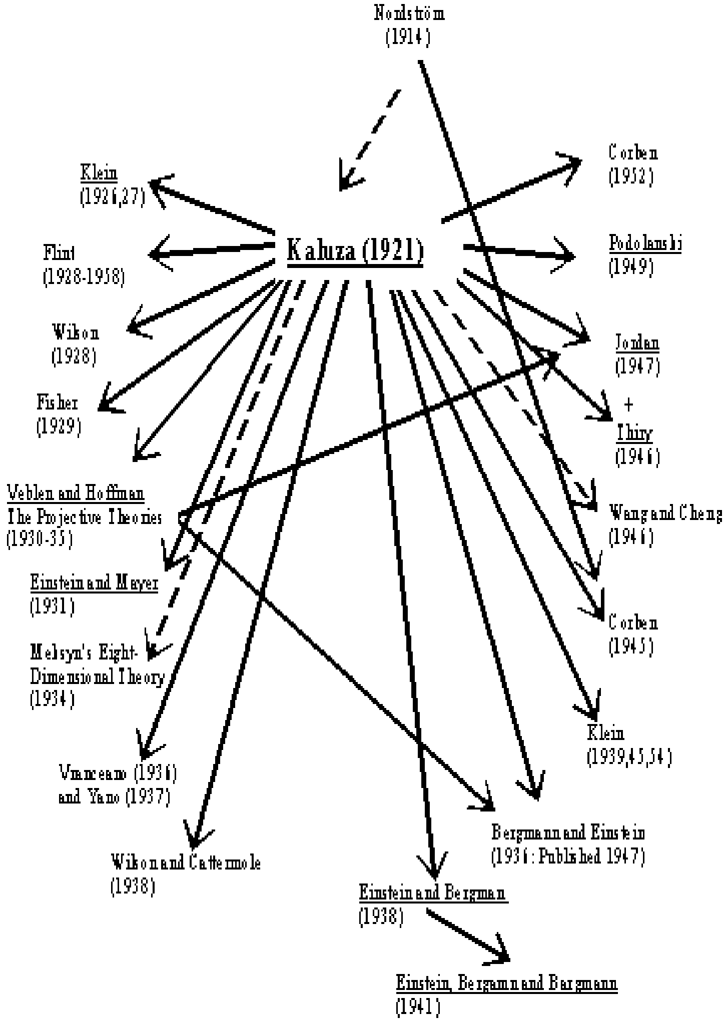
A continual development of the question of whether our space is greater than three dimensions or our space-time is greater than four dimensions, depending on whether the question was asked before or after the advent of the theories of Relativity, has taken shape. Before the General Theory of Relativity there was no physical basis for the possibility of hyper-dimensional spaces, but the suggestion, inherent in the Riemannian geometries, was enough to spur speculations as well as some theoretical research on such theories decades before General Relativity. Some of the speculations went beyond what would normally be considered good science and therefore caused a backlash that meant to deny the possibility of any hyper-dimensional spaces. Besides Riemann, three important figures who have anticipated more modern attitudes on hyper-dimensionality through their own opinions have come forward. Clifford was a precursor of those who hold that space-time can account for all physical phenomena and should be considered as real. Zöllner was a precursor to those who would readily put hyper-dimensional structures to use in either a pseudo-scientific or questionably scientific manner. Attempts such as these cause serious problems for scientists who wish to treat hyper-dimensionality more realistically, because they lend an air of skepticism to the overall concept. And Mach is the precursor of the backlash to hyper-dimensional theories. This last group consists of those who see such theories as mere artificial frameworks, which are unnecessary, at best, and have no relation to objective reality. The opinions espoused by these three men and those that had similar ideas created the atmosphere into which the beginning of both curious speculation and serious scientific thought on hyper-dimensional structures emerged during the last half of the nineteenth century.

In the early twentieth century, the development of General Relativity supplied a physical basis for the possibility of hyper-dimensional structures although General Relativity itself had no need of more than four dimensions to describe curved space-time. Onto this stage Kaluza entered as the originator of the five-dimensional attempts to generalize the General Theory. Nordström had suggested a five-dimensional structure several years earlier, but he was only concerned with the Special Theory of Relativity and his idea remained virtually stillborn and largely unrecognized by the scientific community.

A possible connection between Nordström's ideas and Kaluza's theory cannot be avoided, but the existence of such a connection is speculative at best. However, Nordström's idea does help demonstrate the existence of an awareness within the scientific community of the possibility of five-dimensional theories. Both Kasner and Einstein were subject to this awareness, but their efforts cannot nor should not detract from the claim that Kaluza originated a historical tradition of the development of five-dimensional unified field theories.

Two major lines of succession have emerged from Kaluza's original theory. The main line of succession consists of a group of theories that were developed as extensions and modifications of Kaluza's theory, but are still independent theories in their own right. These theories are well documented in the literature and often cited by other scientists working in the same area of theoretical physics. The other group of theories consists of both independent theories which can stand alone and extensions of Kaluza's theory, usually into the realm of the quantum. These theories are not as well represented in the scientific literature and are in general cited less often and are thus not very well known. Taken together, all of these theories form a more-or-less continuous development of Kaluza's basic hypothesis of a five-dimensional space-time continuum. A tendency may also have emerged whereby the two groups of theories have focused to some extent around two schools of thought centered in London and Paris. The development of these theories also shows other characteristics inherent in the history of science such as the parallel formation of ideas. It is hoped that this historical development demonstrates that speculation on hyper-dimensional spaces is a legitimate scientific endeavor as well as a natural extension of General Relativity. A table, which summarizes the development of ideas after Kaluza in a historical and chronological sequence, follows.

CHRONOLOGICAL COMPARISON OF THE HYPER-DIMENSIONAL THEORIES



The main-line theorists are underlined; An arrow represents a documented historical relationship between theories;
And a broken arrow represents a possible or suspected historical relationship that has not been confirmed.
The chronological order is counter-clockwise beginning in the upper left-hand corner.

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